to influence the experimental results. This list will be an aid in setting up the original experiment and also in carrying the experiment further in case the original analysis shows that the important variables were not covered. In making up this list, the engineer should draw not only on his own technical information and knowledge of the job but also on information which can be contributed by shop supervisors and others. The list should include not only the theoretical variables but also the practical shop-type variables which experience or capability studies have indicated may be present.

In particular, be sure to include the variables which may be associated with processing operations such as cleaning, heat treating, method of stacking material in furnaces or boats, etc. The longer the list of possible variables, the better.

Select from this list four or five variables to be included in the experiment. The experiment will be most successful if the first variables chosen have fairly large effects. While we cannot be sure in advance just what these variables are, there are usually certain reasons to suppose that one variable is potentially more important than another. Attempt to select the variables in the order of their probable importance.

On the other hand, having made every reasonable effort to select important variables at the beginning, do not be unduly concerned about the possibility of choosing the wrong variables. The initial experiment will warn you, by showing a large residual, if there are variables in the process larger than the ones you have selected for study. In that case, go back to the original list, pick out another set of variables which may prove to be productive and set up another experiment.

Your judgment of the variables which should be included will be considerably sharpened after the first experiment is run.

B-8.5 Disposing of the variables not included in the experiment

All the variables which are not to be studied formally must be dropped out statistically so they will be unable to affect the conclusions. There are two methods of doing this:

(1) Randomize the order in which the experimental results are obtained so that any unidentified variables which happen to be present will have an equal chance of affecting all portions of the data. To do this, first set up the experiment in boxes as shown on page 92 and then use some random method of assuring that the boxes will be filled in an unbiased order. Methods of doing this are given in paragraph B-8.6.

(2) If it is convenient and practical, arrange to hold constant some of the variables which are not to be studied. For example, if you do not wish to study the possible effect of various machines, arrange to run the experiment on only one machine. Temperatures, pressures, the composition of chemical baths, etc., can often be held constant or essentially so.

On the other hand, the experimenter should be careful not to assume that variables are constant without sufficient evidence. A single machine may behave differently depending on how it is set up or on its state of repair. The experimenter himself may behave differently according to his physical or emotional state or according to the time of day. We frequently find that conditions which we believed to be constant were actually varying over quite a wide range.

Even where it is possible to hold some of the variables constant, it may be undesirable to do so because of the fact that all of the elements in the going process will not be represented.

B-8.6 Methods of introducing randomness into the experiment

(1) First Method

Throw a die or pair of dice and note the number which comes up. Start counting the boxes in the upper left-hand corner of the design and fill in first the box which corresponds to the number on the dice. This can be indicated by writing the word "first" in the appropriate box. Then throw the dice again and count off more boxes in accordance with the indicated number. Write in this box "second." When you have counted off all the boxes in the design, continue again with the upper left-hand corner but this time skip any boxes which have already been filled. Continue in this manner until all the parts of the experiment have been assigned.

In conducting the experiment, set up first the combination of conditions corresponding to the box marked "first" and run the necessary number of units. Then set up the conditions for the box marked "second" and run the necessary units, etc. This will tend to keep any particular portion of the data from being influenced by the known or unknown variables which come and go in a process with time. It is unlikely,

RANDOM NUMBERS

15	62	38	72	92	03	76	09	30	75	77	80	04	24	54	67	60	10	79	26	21	60	03	48	14
77	81	15	14	67	55	24	22	20	55	36	93	67	69	37	72	22	43	46	32	56	15	75	25	12
18	87	05	09	96	45	14	72	41	46	12	67	46	72	02	59	06	17	49	12	73	28	23	52	48
08	58	53	63	66	13	07	04	48	71	39	07	46	96	40	20	86	79	11	81	74	11	15	23	17
16	07	79	57	61	42	19	68	15	12	60	21	59	12	07	04	99	88	22	39	75	16	69	13	84
54	13	05	46	17	05	51	24	53	57	46	51	14	39	17	21	39	89	07	35	47	87	44	36	62
95	27	23	17	39	80	24	44	48	93	75	94	77	09	23	48	75	91	69	03	55	51	09	74	47
22	39	44	74	80	25	95	28	63	90	41	19	48	46	72	51	12	97	39	83	35	83	23	17	29
69	95	21	30	11	98	81	38	00	53	41	40	04	16	78	67	29	83	41	18	30	90	44	37	64
75	75	63	97	12	11	57	05	86	52	82	72	47	72	14	37	72	69	75	48	72	21	52	51	81
08	74	79	30	80	70	11	66	79	25	88	01	94	52	31	38	57	98	71	62	12	56	61	01	54
04	88	45	98	60	90	92	74	77	87	40	18	65	87	37	08	68	62	39	52	84	74	90	68	18
97	35	74	05	75	42	13	49	48	38	74	19	06	42	60	20	79	90	81	77	18	51	71	27	27
53	09	93	28	29	80	19	68	30	45	94	49	49	71	21	93	93	71	30	34	52	65	83	40	13
26	36	68	48	09	37	69	26	22	80	23	34	10	45	70	83	51	07	37	44	62	96	74	42	64
49	16	57	15	79	56	63	22	94	28	11	39	69	55	38	53	06	97	20	42	09	14	90	43	48
03	51	79	78	74	75	23	73	75	98	47	85	07	26	02	61	28	01	22	16	14	12	15	67	22
21	88	87	28	48	23	44	03	03	80	53	89	07	87	93	30	17	84	17	74	16	53	31	39	01
56	41	73	33	41	59	16	59	50	98	24	24	87	06	75	99	52	09	88	05	86	25	43	50	94
72	39	19	70	17	01	04	01	22	33	04	84	63	27	65	84	39	45	55	31	95	88	93	90	37
97	28	25	81	49	71	69	22	04	51	56	46	56	15	10	69	59	99	50	29	33	50	16	93	09
18	87	02	72	08	74	52	16	03	82	20	19	66	23	62	37	51	04	89	31	32	19	59	85	57
53	40	11	75	45	13	56	85	31	37	09	17	71	96	79	39	50	79	27	62	71	14	95	53	03
60	49	03	41	56	78	33	77	28	92	21	90	10	62	01	97	06	45	01	19	95	12	24	18	52
09	16	12	75	04	39	69	95	00	48	26	85	28	73	08	66	92	10	66	75	62	61	27	82	57
64	20	19	87	54	88	15	12	54	24	06	99	57	07	28	51	34	54	98	50	70	88	02	86	48
31	28	07	58	77	03	98	26	76	09	10	44	57	61	28	60	29	85	70	79	80	29	19	98	92
80	04	28	47	76	35	73	67	78	28	09	39	88	63	74	41	26	92	42	33	06	80	06	33	84
24	60	22	51	19	34	54	08	24	73	86	72	11	44	69	76	90	81	17	85	57	47	35	16	84
59	16	11	26	29	18	97	78	44	43	58	92	78	70	80	09	65	32	68	26	65	73	90	50	46
58	54	29	98	27	40	51	92	07	13	58	41	59	56	94	16	32	51	42	54	77	37	13	85	19
20	18	34	22	73	57	40	67	17	28	63	57	74	36	18	65	55	25	50	68	35	90	00	03	38
53	90	46	56	19	50	58	33	84	53	14	74	17	40	73	86	11	04	02	04	02	28	49	62	36
97	16	93	94	65	70	95	95	83	20	91	42	57	95	63	00	86	29	02	53	02	27	86	70	95
72	55	71	70	92	04	22	53	19	29	67	29	13	56	70	45	73	45	05	04	32	43	30	93	41
99	19	72	58	35	49	09	26	00	74	26	42	94	52	02	83	31	85	65	66	31	97	67	52	15
48	21	49	72	97	79	19	64	81	82	78	92	51	96	51	28	79	13	20	82	34	81	39	46	86
52	37	68	15	53	22	98	30	16	31	83	24	87	69	29	24	85	44	25	50	75	62	83	95	41
97	50	52	53	52	26	78	21	68	69	57	79	42	40	89	55	81	75	24	52	51	32	79	97	05
36	05	09	18	11	71	01	63	17	60	11	65	19	43	07	44	86	19	58	92	23	71	32	96	19
20	79	70	09	30	81	14	53	80	93	71	94	10	18	14	83	69	76	53	25	27	36	65	65	05
13	07	89	72	08	00	37	75	14	94	83	85	06	72	66	07	47	30	17	11	16	02	63	97	30
94	26	82	37	43	34	23	00	14	50	96	85	41	17	71	69	20	15	98	82	79	69	68	50	31
13	55	88	38	43	75	37	43	83	85	53	74	54	62	99	68	93	74	43	95	06	26	79	78	87
02	44	24	97	71	97	93	12	70	89	42	52	33	24	91	05	87	53	15	77	49	92	83	97	80

Fig. 110. Table of random numbers.

for example, that one of the unknown variables would happen to affect all of the data under factor A1 and none of the data under factor A2.

(2) Second Method

Write on cards or slips of paper all the combinations of variables to be represented in the experiment. For example:

A1B1C1D1
A1B1C2D1,etc.

Shuffle the cards thoroughly and put them together to form a deck. Set up first the combination of conditions shown on the top card in the deck and run the necessary number of units. Then continue with the next card and so on. This will make sure that the data are obtained in random order and will help to keep unanticipated variables from introducing an unexpected bias.

(3) Third Method

Use a table of random numbers to determine the order in which the portions of the experiment should be run. A typical page from one of the published tables is shown in Figure 110. The table is used as follows:

> Starting at random at any point in the table, take the first digit you find and count off that number of boxes in the experimental design. Write the word "first" in the indicated box. Then take the digit directly below and continue in this manner, using the digits in exactly the same way as the numbers which came up on the dice in Method (1).
>
> The table of random numbers can also be used in other ways. Take the digits vertically or horizontally. Take the first column in the separated blocks of five columns or alternatively take the last column, the fourth column, etc. Take either the last digit, the next to the last digit, pairs of two digits in combination, and so on. It is also possible to take the numbers in diagonal rows: for example, 15, 81, 05, 63, etc. in Figure 110. One can start at the bottom and read up, start at the right-hand side and read across toward the left, skip numbers in any way desired, taking every fourth pair, every tenth individual digit, etc.
>
> Because of the large number of ways in which the table can be used, a single table of the type shown in Figure 110 can be used indefinitely without introducing nonrandom patterns into the data.

Other methods of ensuring randomness in the results may occur to the engineer. He should avoid, however, attempting to make a "hit-and-miss" or haphazard assignment of the boxes. Because of the strong psychological tendency of a human being to repeat patterns, it is virtually impossible to ensure randomness without the aid of something which is dependent directly on the laws of chance. For example, a truly random set of numbers will repeat digits in succession, or alternate even and odd digits, more frequently than a person who is attempting to give numbers at random.

B-8.7 Methods of handling abnormal data

Experiments are frequently disturbed by the accidental loss of a unit which was to have been used for one of the boxes. Where possible, the experimenter should forestall this possibility by running more units than will be essential for the experiment and select from this group at random as described on page 113. Occasionally, however, it may be so expensive to produce the units or obtain measurements that provision for additional units is out of the question. In that case, the engineer should do one of the following things to take care of the gap in the data:

(a) Omit the measurement completely and leave a gap in the plotting.

(b) Fill in an arbitrary value if necessary by calculations based on other numbers in the experiment. See Reference No. 5.

A second common experience is the obtaining of a measurement which looks like a "freak." This should be handled as in the case of any data in a process capability study. (See pages

52–53). The experimenter should label the measurement in such a way as to identify it but should not, in general, attempt to eliminate it from the data.

In handling freaks on experimental charts, keep in mind the following points:

(1) In view of the small total quantity of data, each single measurement carries a large proportion of the information. Do not overlook the possibility that certain combinations of variables may tend to produce the condition you are tempted to call a "freak."

(2) Since the data in a designed experiment are arranged and rearranged many times in order to study the different factors in different combinations, a single freak is likely to appear in several different ways. Be careful not to conclude from this that the entire process is full of freaks.

B-8.8 Protecting the Identification of the experimental units

Since all the conclusions which will be obtained from the experiment depend upon careful and precise identification of the measurements, the experimenter must take constant precautions to preserve the necessary identification. Ordinarily at least part of the processing and handling will be done by the shop, or by other people who are not directly responsible for the experiment. An inadvertent mixup in the units, or the processing of the A1B2 units at the temperature which was planned for A1B1, may make it impossible to obtain useful conclusions.

In all cases, the experimenter should either follow through all units in the experiment himself, or make sure that others who are doing this for him have been carefully instructed. Precautions of this kind invariably pay off in more reliable and more conclusive results.

PART C
Specifications

C-1 SPECIFICATIONS IN GENERAL

In manufacturing processes, we are interested in the characteristics of each and every unit produced. Even when we attempt to study the process by means of samples, as in process capability studies or shop control charts, we are really interested in the total distribution of individuals and are using the samples as a means to this end. Specifications are stated by design engineers or product engineers in an attempt to set up desirable restrictions on:

(a) the individual units, or
(b) the distribution of individual units,

or both.

Specifications tend to fall into three basic types.

Type A. The specification states a limit or other requirement which applies to each unit of product individually regardless of other units in the same product. For example: "The length of the part shall be .125″ ± .003″." "The width of the groove shall not exceed .375″." Product is considered to conform to such specifications if each individual unit is on or inside the limit, even if all units are exactly at the limit.

Type B. The specification defines the distribution which the product must have in order to be acceptable. For example: "The average of the product shall not be higher than .5 millivolt and the individual units shall be distributed in a natural manner around this average with a spread not to exceed ±.03 millivolt." Such specifications are sometimes spoken of as "distribution requirements." They may or may not be accompanied by limits of Type A which apply to the individual pieces.

Occasionally, requirements of this type are specified in terms of $\bar{X}$ and R charts. That is, the specification states the centerline and control limits for an $\bar{X}$ and R chart and the product is acceptable as long as random samples from the product show control on this chart.

Type C. The specification states a requirement which must be met by most of the product but allows a certain percentage of units to exceed the requirement. For example: "The resistance shall not exceed 173 ohms. However, product shall be considered acceptable if not more than 2% exceeds this limit provided no units exceed 178 ohms."

Such requirements are sometimes referred to as "product tolerance" requirements. When the Government specifies the AQL which a given product must meet, this is in effect a Type C specification.

On most products the majority of specifications are of Type A. However, the number of Type B specifications is gradually increasing. The engineer should be aware that many specifications are stated as if they were of Type A and yet the designer has in mind a distribution which he expects the product to meet. Such requirements are, in the mind of the designer, specifications of Type B.

In a quality control program there are many advantages in working with Type B specifications.

C-2 RELATIONSHIP BETWEEN PROCESS AND SPECIFICATION

To make a valid comparison between a process and a specification, it is necessary to have an $\bar{X}$ and R chart with both $\bar{X}$ and R in control. Follow the directions on page 56 to find how the process distribution is related to the specified limits. If necessary, make calculations as shown on pages 58–60 in order to determine, more or less accurately, how much of the distribution can be expected to fall outside of limits. There are four basic relationships which may exist between the process and its specifications, as follows.

(1) *The spread of the process may be less than the difference between the specified maximum and minimum, with the process safely centered.* See Figure 111.

Possible action:

a. Maintain control against these standard values.
b. Consider the use of modified control limits for shop charts as explained on pages 195–196.
c. Consider the possibility of reducing inspection as explained on page 274.

(2) *The spread of the process may be just equal to the difference between the specified maximum and minimum.* See Figure 112.

Possible action:

a. Provide for constant checking of the process to keep it centered.
b. Provide for sorting the product when the distribution shifts.
c. Attempt to reduce the process spread through a designed experiment.
d. If possible, get wider specifications.

(3) *The spread of the process may be less than the difference between the specified maximum and minimum, but the process may be off-center.* See Figure 113.

Possible action:

a. Try to center the distribution at a point safely within the specified limits. Maintain control at that point.
b. If the shop is unable to center the distribution within limits, and if the present level does not produce a good product, write this down as *unfinished business.* Put a control chart in the shop and study

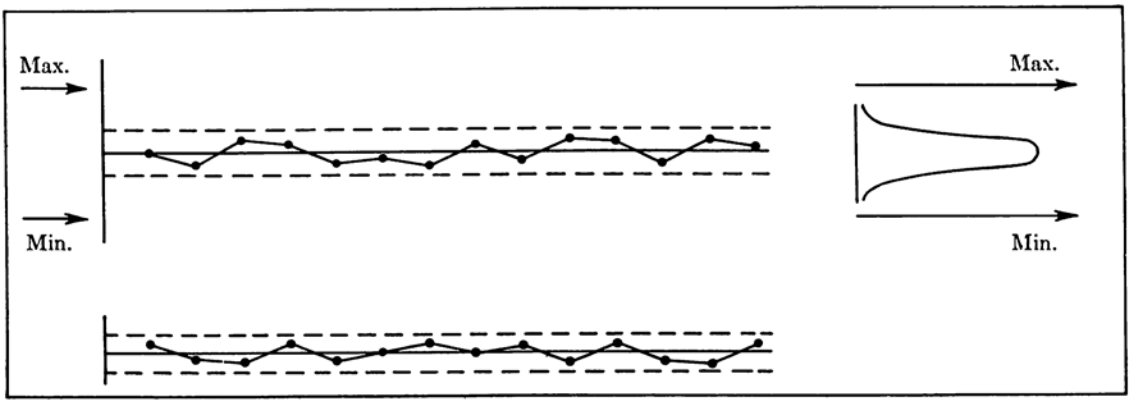


Fig. 111. Process narrower than specified limits.

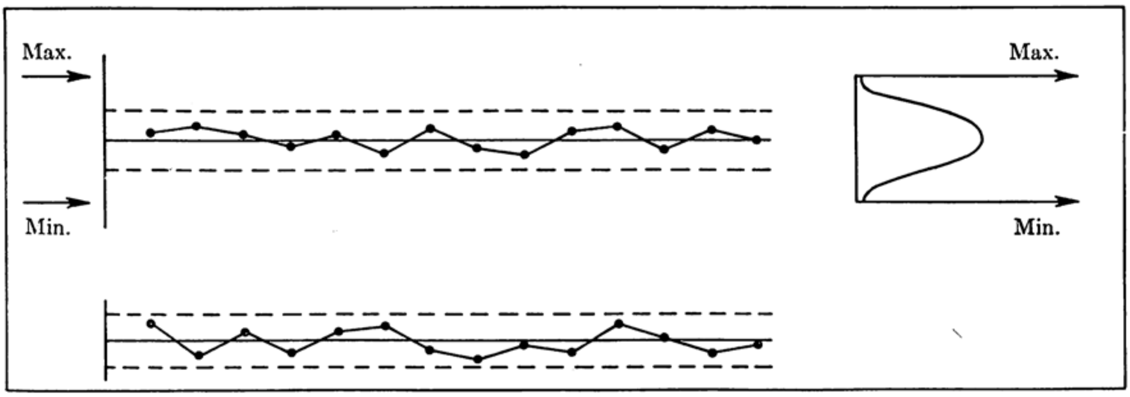


Fig. 112. Process spread equal to specified limits.

it regularly at your quality control meetings. If necessary, design an experiment to discover what can be used to make the process shift.

Meanwhile provide for operational sorting until the necessary information is obtained. Don't give up until you get this problem solved.

c. Determine whether the specification nominal can be shifted without adverse effect on the product. If so, take steps to get the specification changed.

It sometimes happens that a distribution cannot be shifted to meet one specification without causing failures to meet another specification. In that case, there may be correlation between the characteristics, and the two specifications may be incompatible. Put a control chart on each characteristic and study the charts together. Include both characteristics in any designed experiments which are conducted. This will make it possible to find the optimum combination of distributions and maximize yields on both characteristics simultaneously.

(4) *The spread of the process may be greater than the difference between the specified maximum and minimum.* See Figure 114.

Possible action:

a. Try to open the specifications.
b. Try to reduce the spread of the process by running a designed experiment.
c. Provide for 100% sorting of all product until the problem can be resolved.
d. Aim for a level that will set an economic balance between relative costs, including rework or scrap. Maintain control at that level.
e. Make fundamental changes in the process, such as: buy a new machine, design different tools or provide different methods.

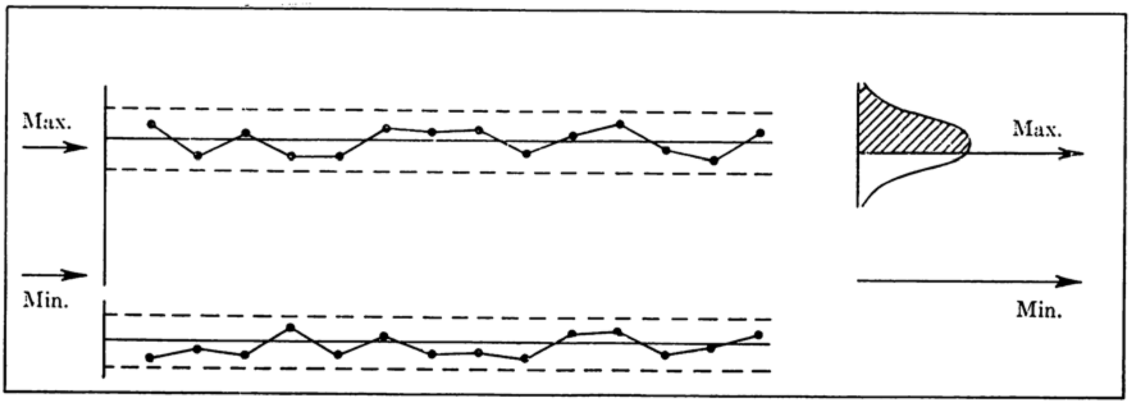


Fig. 113. Process offcenter.

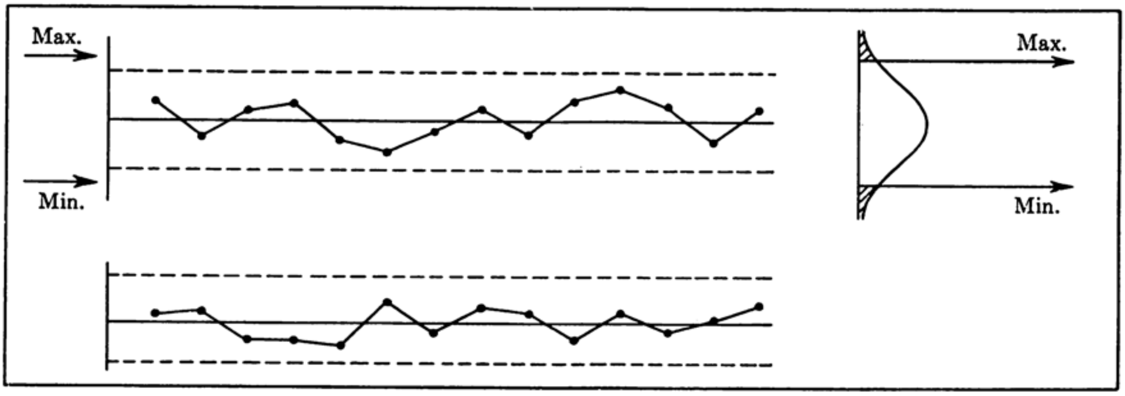


Fig. 114. Process wider than specified limits.

Non-normal distributions

The foregoing comparisons apply to any distribution, whether normal or not. However, if the distribution is not normal, the center of the distribution may need to be located closer to or farther from one of the specification limits in order to allow for the non-normal shape.

C-3 SPECIFICATION CONFLICTS AND WHAT CAN BE DONE TO AVOID THEM

If $\bar{X}$ and R charts show that the natural distribution of the process is too wide to fit between the specified limits (Condition 4), or if the process cannot be centered in the proper place (Condition 3), there is evidently a conflict between process and specification. As indicated above, there are three ways in which this conflict may be resolved:

(1) Change the process.

(2) Change the specification.

(3) Sort and repair the product which falls out of limits.

The first thing to attempt is to change the process. If making the necessary changes would be very expensive, or if no better way of making the product is known, the engineers should look carefully at the specification. The usual procedure is to ask to have the tolerances reviewed to see what effect a different set of tolerances would have on the assembly, functioning or interchangeability of the product.

In order to reduce manufacturing costs to a minimum, tolerances should be as wide as the design can permit rather than as narrow as the shop is able to meet.

Sorting and repair is a very expensive way of handling specification trouble, and should only be considered as a last resort.

The following will be helpful to engineers in avoiding unnecessary costs resulting from specification conflicts:

(1) The natural spread of a process is usually taken to be $\pm 3\ \sigma'$. For economical manufacture, the specified tolerances should accept the full natural process spread.

(2) In addition, it is seldom possible to keep a process running at exactly the same level month after month. Some allowance is needed to permit a slight shifting of the center. For most processes, it is considered desirable to allow the center to shift about $\pm 1\ \sigma'$. In that case, the specified tolerances should be about $\pm 4\ \sigma'$.

In some Western Electric processes where good control around the nominal is essential, the specified tolerance is $\pm 3^1/_3\ \sigma'$. This permits the process average to shift up or down about $^1/_3\ \sigma'$.

(3) If the natural spread of the process is more than about $^2/_3$ of the specified tolerances, it is probably great enough to cause occasional difficulty in meeting the requirements. If the spread is considerably less than the specified tolerances (say $^1/_2$ or $^1/_3$), it may be possible to reduce costs by using a more economical process.

In all cases where processes are to be compared with specifications, the processes must first be in control as shown by a process capability study. See paragraph A-3.10 on page 56. If an out-of-control condition is indicated by the study, identify the assignable causes and remove them (or allow for them) before comparing the process with the specification.

C-4 STATISTICAL ADDITION OF TOLERANCES

Whenever two or more parts are assembled together, the act of assembly creates new dimensions and new distributions that did not exist before the assembly was made. The engineer is interested in predicting the characteristics of the assembly, and in assigning suitable tolerances to the components so as to permit the most economical and trouble-free manufacture for both the components and the assembly.

Every assembly problem of this nature involves the *addition of distributions.* The distributions which exist on one component are added to the distributions which exist on the second component and so on until the assembly is completed.

Inasmuch as the addition of distributions is a statistical procedure, the engineer should be familiar with certain basic statistical laws in order to arrive at economical solutions.

C-4.1 Theory of the addition of distributions

The most important statistical laws which govern the addition of distributions are the following:

(1) *Law of the addition of averages.* If parts are assembled in such a way that one dimension is added to another, the average dimension of the assembly will be equal to the sum of the average dimensions of the parts.

Let $\bar{X}_A$ = the average dimension of Part A
$\bar{X}_B$ = the average dimension of Part B
$\bar{X}_C$ = the average dimension of Part C, etc.

Average dimension of assembly = $\bar{X}_A + \bar{X}_B + \bar{X}_C$, etc.

See Figure 115.

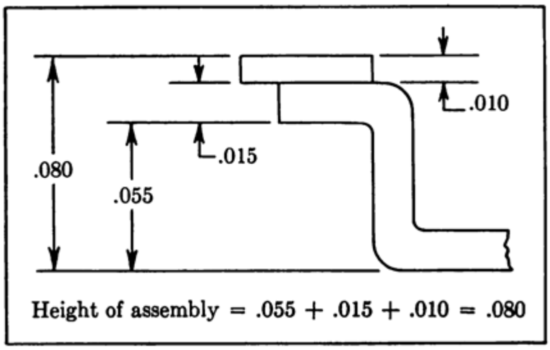


Fig. 115. Addition of averages.

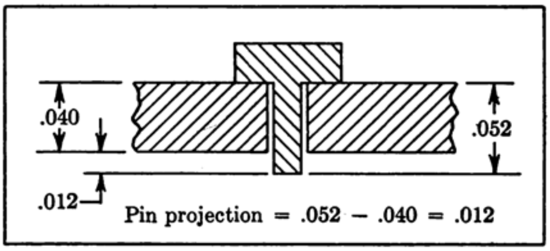


Fig. 116. Subtraction of averages.

(2) *Law of differences.* If the parts are assembled in such a way that one dimension is subtracted from another, the average dimension of the assembly will be the difference between the average dimensions of the parts.

Let $\bar{X}_D$ = the average dimension of Part D
$\bar{X}_E$ = the average dimension of Part E

Average dimension of assembly = $\bar{X}_D - \bar{X}_E$ or $\bar{X}_E - \bar{X}_D$ as the case may be.

See Figure 116.

(3) *Law of sums and differences.* If the parts are assembled in such a way that certain dimensions are added to each other while certain dimensions are subtracted, the average dimension of the assembly will be the algebraic sum of the average dimensions of the parts.

Average dimension of assembly = $\bar{X}_A + \bar{X}_B + \bar{X}_C - \bar{X}_D + \bar{X}_E$, etc.

(4) *Law of the addition of standard deviations or variances.* If the components are assembled at random, the standard deviation of the assembly will not be the simple sum of the standard deviations of the parts. It will be the value obtained by squaring each of the component standard deviations, totaling the squares, and then taking the square root of the total.*

Let σ_A = the standard deviation of Part A
σ_B = the standard deviation of Part B

Standard deviation of the assembly $\sqrt{(\sigma_A)^2 + (\sigma_B)^2}$.

The fourth law should be carefully studied by the engineer, because the statistical addition gives a different result from the one which he would be likely to obtain intuitively.

In particular, the engineer should note that

* In special cases, where the dimensions do not combine linearly, or are not independent, more complicated calculations may be necessary to obtain the final dimension and its standard deviation.

the squares of the standard deviations are always *added* regardless of whether the average dimension is obtained by sums or differences. Never attempt to subtract one standard deviation from another as may be done in the case of averages.

The fourth law can also be expressed in terms of "variance" instead of standard deviation. The variance is the square of the standard deviation (σ^2). If $(\sigma_A)^2$ is the variance of Part A and $(\sigma_B)^2$ is the variance of Part B, the variance of the assembly will be $(\sigma_A)^2 + (\sigma_B)^2$.

C-4.2 Assembly tolerances

The law of the addition of standard deviations has important implications in assembly work, since the "square root of the sum of the squares" will always be less than the value that would be obtained if the standard deviations were merely totaled. For example:

$$\sigma_A = .0003$$

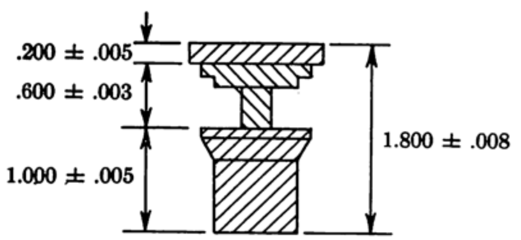
$$\sigma_B = .0004$$

$$\sigma_A + \sigma_B = .0007$$

But:

$$\sqrt{(\sigma_A)^2 + (\sigma_B)^2} = \sqrt{(.0003)^2 + (.0004)^2} = \sqrt{.00000025} = .0005$$

The law of statistical addition gives .0005 while simple arithmetic addition gives .0007. This means that *random assemblies can be held to narrower spreads than would be indicated by totaling the spreads of the parts.* Designers take advantage of this in the condition known as "overlapping tolerances."



The parts may be ±.005, ±.003 and ±.005, but the assembly is only allowed to spread ±.008.

Fig. 117. Overlapping tolerances.

C-4.3 Overlapping tolerances

When we add up the tolerances on all component parts on a drawing and find that their total is greater than the tolerance allowed for the assembly, we have a condition known as "overlapping tolerances." See Figure 117.

This may or may not be a cause for concern since, by the law of the addition of standard deviations as given above, we know that the spread of random assemblies will be less than the total spread of all the parts.

Whether we will have trouble in assembly when using these "overlapping tolerances" depends on four factors:

(1) Whether the actual standard deviations of the components are really the same as implied in the tolerances on the drawing.

(2) Whether the actual averages of the components are the same as the nominals shown on the drawing.

(3) Whether the components are assembled at random.

(4) Whether the "square root of the sum of the squares" of the actual standard deviations, when calculated, is compatible with the tolerance specified for the assembly.

The information for points 1 and 2 must be obtained from process capability studies or shop control charts. Point 3 can be taken care of in setting up the assembly process. Point 4 can be calculated by the engineer from the information provided in 1 and 2.

C-4.4 Pitfalls in the use of overlapping tolerances

The law of the addition of standard deviations as given on page 123 applies in any case where the standard deviations of the various components are known. Engineers sometimes wish to take advantage of this law without having prior knowledge of what the standard deviations are likely to be. In such cases the engineer may reason as follows:

a. Assume that the components will all be normally distributed with a spread equal to the tolerance which is put on the drawing.

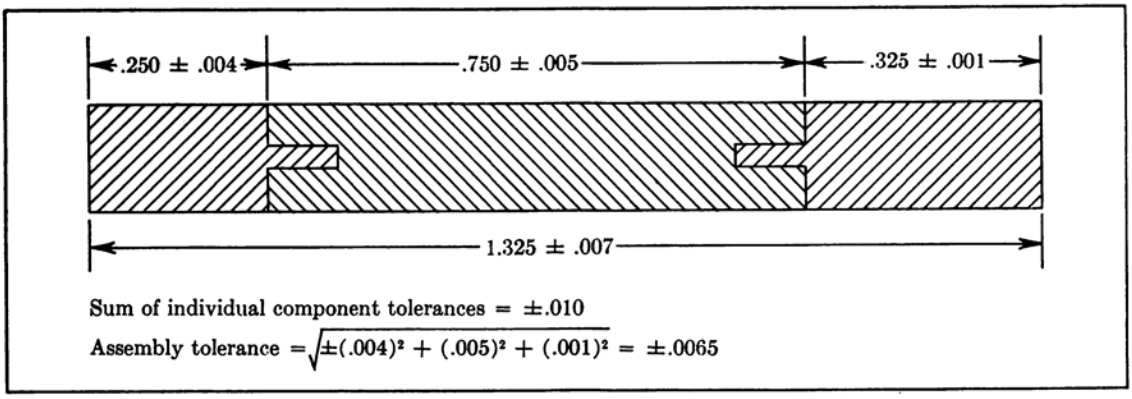


Fig. 118. Wrong use of overlapping tolerances.

b. It would then be possible to substitute "specified tolerance" in place of "standard deviation" in the equation on page 123.

Figure 118 shows an example of assembly limits calculated in this manner.

The calculations were as follows:

Average of assembly should equal $\text{Nominal}_A + \text{Nominal}_B + \text{Nominal}_C$

$$.250 + .750 + .325 = 1.325.$$

Tolerance of assembly should equal

$$\sqrt{(\text{Tolerance}_A)^2 + (\text{Tolerance}_B)^2 + (\text{Tolerance}_C)^2}$$

$$\sqrt{(.004)^2 + (.005)^2 + (.001)^2} = \sqrt{.000042} = .0065$$

On the basis of the above calculation the engineer fixed the assembly tolerance at ±.007. This is a dangerous way to use overlapping tolerances.

The danger in making calculations of this type is that the engineer has no way of checking his assumptions. If the components are not normally distributed around nominal, or if their spread is not equal to the tolerance, the shop may get into serious trouble when tolerances are calculated in this manner.

In particular, the shop will be likely to get into trouble if the spread of the process at any one time is considerably narrower than the tolerance. This is very likely to be the case in practice. To avoid such trouble the engineer should adopt the following rules.

(1) Always calculate the assembly tolerance from process capability information. If no information is available, use your best estimate of the probable capability and make ample allowance for the fact that your estimate may be inaccurate.

(2) In using specifications which include overlapping tolerances, always provide the shop with control charts which will show the actual distribution of the components. The assemblies will be the statistical sum of the distributions on the control charts, and this may be very different from the assumed distributions on the drawing.

C-4.5 Successful use of overlapping tolerances

Figure 119 shows an example of the successful use of overlapping tolerances, properly implemented with control charts.

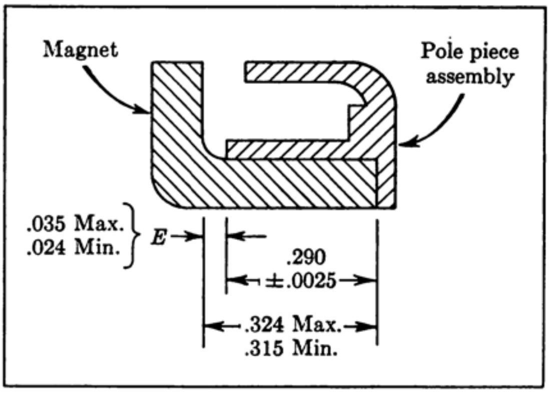


Fig. 119. Correct use of overlapping tolerances.

The "E" requirement on the gap is only .035 – .024. The possible maximum clearance using extreme parts would be .324 – .2875 = .0365. The possible minimum clearance using extreme parts would be .315 – .2925 = .0225. Yet the shop consistently meets the tight "E" requirement.

Procedure

- The .290″ dimension is controlled by an $\bar{X}$ and R chart.
- The .324 maximum, .315 minimum dimension is controlled by an $\bar{X}$ and R chart.
- The .035 maximum E, .024 minimum E dimension is verified by an $\bar{X}$ and R chart.

The results have been economical manufacture, minimum inspection, virtually no rejections and good process control.

C-5 CLEARANCES AND FITS

"Statistical addition of tolerances" can be applied to advantage in dealing with clearances and fits. These are "mating" conditions expected of two or more parts having the same or different tolerances. The specified mating conditions vary from interference fits to running fits according to the functional design of the mating parts. See Figures 120 and 121.

Taking Figure 121 as an example, what will be the average clearance between shaft and bearing, and how much will it vary?

Let $\bar{\bar{X}}_B$ = the controlled average of the inside diameter of the bearing, and let σ_B = its standard deviation.

Let $\bar{\bar{X}}_S$ = the controlled average of the outside diameter of the shaft, and let σ_S = its standard deviation.

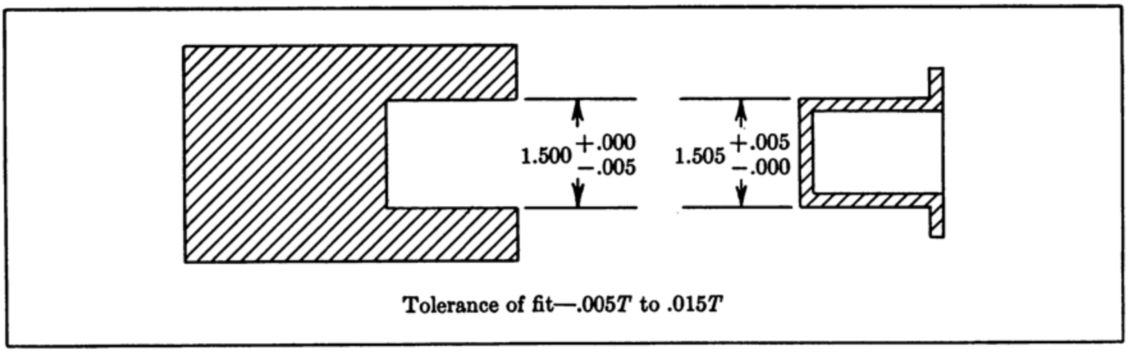


Fig. 120. Tight clearance.

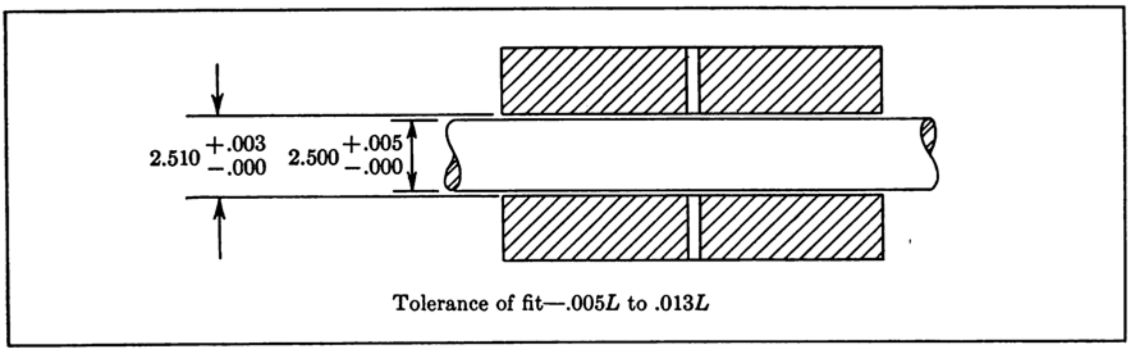


Fig. 121. Loose clearance.

A process capability study has yielded the following values:

$$\bar{\bar{X}}_B = 2.5115 \qquad \bar{\bar{X}}_S = 2.502$$

$$\sigma_B = .0006 \qquad \sigma_S = .0007$$

The evidence indicates that the distributions are approximately normal.

Average assembly clearance

$$= \bar{\bar{X}}_B - \bar{\bar{X}}_S$$

$$= 2.5115 - 2.502$$

$$= .0095$$

Standard deviation of the assembly clearance

$$(\sigma_C) = \sqrt{(\sigma_B)^2 + (\sigma_S)^2}$$

$$= \sqrt{(.0006)^2 + (.0007)^2}$$

$$= \sqrt{.00000085}$$

$$= .0009$$

Minimum clearance $= .0095 - 3\ \sigma_C = .0095 - .0027 = .0068$.
Maximum clearance $= .0095 + 3\ \sigma_C = .0095 + .0027 = .0122$.

To find how much of the product can be expected to meet the specification (assuming normal distributions), proceed as follows:

$$t = \frac{.0095 - \text{Spec. Max.}}{\sigma_C} = \frac{.0095 - .013}{.0009} = -3.9$$

Look up -3.9 in Table I on page 133, under "Percentage Outside of Max." Only one-hundredth of 1% of the product is likely to fail to meet the maximum tolerance. Similar calculations are made for the minimum tolerance. Follow the directions on page 132.

Comment

In order to take advantage of statistical solutions for such problems, two restrictions must be met:

(1) There must be a known distribution for each component. This ordinarily means that each component must come from a controlled process, or at least from a known process which is covered by $\bar{X}$ and R charts.

(2) The mating components must be assembled at random rather than by selection.

As in the case of overlapping tolerances, the engineer should *not* use the nominals or tolerances specified on the drawing. It is necessary to use the actual $\bar{\bar{X}}$'s and standard deviations from controlled processes, as shown by a shop control chart or a process capability study.

PART D
Distributions

D-1 CHARACTERISTICS OF FREQUENCY DISTRIBUTIONS

Frequency distributions have three characteristics that provide useful information:

(1) Center, or average.
(2) Spread, or dispersion.
(3) Shape.

Each of these characteristics can be described by means of standard statistical measures.

D-1.1 Center or average

When observations are plotted in the form of a frequency distribution they usually tend to cluster near some central value, with fewer readings falling on either side. The point near which the measurements tend to cluster is called the "central tendency." Among the common measures of central tendency are the following:

(1) Arithmetic mean (commonly spoken of as "average").
(2) Median (or middlemost value).
(3) Mode (the value having the highest frequency).

The arithmetic mean or "average" is almost universally used in quality control. In a few cases the median is employed as a convenient substitute.

Arithmetic mean

This is denoted by the symbol $\bar{X}$. It is calculated as follows:

Add the observed values and divide the total by the number of observations.

Let X = an individual observation

$$\bar{X} = \frac{X_1 + X_2 + X_3 + X_4 \ldots\ldots X_n}{n}$$

Median or middlemost value

This is denoted by the symbol MED. It is calculated as follows:

Arrange the measurements in ascending order of magnitude. Count off equal numbers of measurements from either end of the series until (a) a single value or (b) a pair of values is left at the center. If a single value is left, this value is the median. If two values are left, the median is the average of the two. For example:

a. 72 79 80 81 93
MED = 80.

b. 46 [51 54] 60
MED = the average of 51 and 54, or 52.5.

Mode

The mode is often used in referring to a skewed distribution. It represents the maximum point on the distribution curve. In a skewed distribution the median and the mode do not occur at the same point as the arithmetic mean.

D-1.2 Spread or dispersion

The "spread" of a distribution is the amount of variation or dispersion of the individual values around their average. Among the common measures of dispersion are the following:

(1) Variance (the mean square deviation of the values from their average).
(2) Standard deviation (the square root of the variance, or "root mean square" deviation of the values from their average).
(3) Range (the difference between the highest and lowest value in a set of observations).

All three of these measures are employed in quality control.

Variance

This is denoted by the symbol σ^2 (sigma squared). It is calculated as follows:

Obtain the average of the given values. Calculate the difference between each value and the average. Square these differences, total them, and divide by the number of given values.

Let X = an individual value

$\bar{X}$ = the arithmetic mean (or average) of the individual values

$$\sigma^2 = \frac{\Sigma[(X - \bar{X})^2]}{n}$$

Example:

Given the set of numbers:

10 14 6 2

$\bar{X} = \frac{32}{4} = 8$

Deviations from $\bar{X}$ are $+2$, $+6$, -2, -6. Squares of deviations are 4, 36, 4, 36.

Mean square deviation (or variance)

$$= \frac{4 + 36 + 4 + 36}{4} = 20$$

Standard deviation

The standard deviation is denoted by the symbol σ (sigma) or sometimes s. It is calculated as follows:

Obtain the variance as directed above. Extract the square root. This is the standard deviation.

$$\sigma = \sqrt{\frac{\Sigma[(X - \bar{X})^2]}{n}}$$

Example:

Given the set of numbers:

10 14 6 2

$\bar{X} = 8$

$$\sigma = \sqrt{\frac{(+2)^2 + (+6)^2 + (-2)^2 + (-6)^2}{4}}$$

$$= \sqrt{20} = 4.472$$

In calculating the standard deviation of a large number of measurements, it is convenient to group the data into cells as shown on page 139. The standard deviation can then be calculated by a short-cut method as shown in Figure 122.

Proceed as follows:

(1) Record the observed frequency opposite the midpoint of each cell.

(2) Set up some convenient arbitrary scale which can be used for preliminary calculations.

(3) Fill in the columns "fx" and "fx^2" as indicated.

Data from Figure 130 on page 139.
$n = 100$

Midpoint of Cell	Observed Frequency, f	Arbitrary Scale, x	fx	fx^2
12.75	1	+10	+10	100
12.55	0	+ 9	0	0
12.35	3	+ 8	+24	192
12.15	1	+ 7	+ 7	49
11.95	3	+ 6	+18	108
11.75	2	+ 5	+10	50
11.55	4	+ 4	+16	64
11.35	10	+ 3	+30	90
11.15	10	+ 2	+20	40
10.95	8	+ 1	+ 8	8
10.75	11	0	0	0
10.55	7	− 1	− 7	7
10.35	4	− 2	− 8	16
10.15	15	− 3	−45	135
9.95	6	− 4	−24	96
9.75	6	− 5	−30	150
9.55	2	− 6	−12	72
9.35	3	− 7	−21	147
9.15	0	− 8	0	0
8.95	0	− 9	0	0
8.75	0	−10	0	0
8.55	1	−11	−11	121
8.35	3	−12	−36	432
	100		−51	1877
Divided by n			−.51 E1	18.77 E2

A = midpoint of zero cell = 10.75

m = cell interval (difference between midpoints) = .20

$\bar{X} = A + m\,(E1) = 10.75 + [.20 \times (-.51)] = 10.75 - .10 = 10.65$

$\sigma = m\sqrt{E2 - (E1)^2} = .2\sqrt{18.77 - .26} = .2\sqrt{18.51} = .86$

Fig. 122. Short method of calculating the standard deviation from a grouped frequency distribution.

(4) Divide both the "fx" and the "fx^2" columns by n (the total number of observations). Call these respectively E1 and E2.

(5) Record the value of A (the midpoint of the cell called "0" on the arbitrary scale) and m (the difference from midpoint to midpoint of the cells).

(6) Calculate $\bar{X}$ and σ for the grouped frequency distribution by using the equations given at the bottom of Figure 122.

Other methods of calculating the standard deviation can be found in the standard statistical texts.

Standard deviation of a universe (σ')

When we wish to refer to the standard deviation of an underlying universe or parent population, we use the symbol σ'. In an industrial process the true value of σ' is usually unknown. However, it is possible to estimate σ' by using a sample (or series of samples) as follows:

$$\sigma' = (\text{sigma of a sample of given size}) \times \frac{1}{c_2}$$

where c_2 is a factor which varies with sample size as shown in Figure 123. σ' can also be estimated from the centerline on an R chart as follows:

If the R chart shows control,

$$\sigma' = \frac{\bar{R}}{d_2}$$

where d_2 is a factor which varies with sample size as shown in Figure 123.

Sample Size	d_2	c_2
2	1.128	.5642
3	1.693	.7236
4	2.059	.7979
5	2.326	.8407
6	2.534	.8686
7	2.704	.8882
8	2.847	.9027
9	2.970	.9139
10	3.078	.9227

Fig. 123. Table of factors for estimating σ'.

In cases where the R chart is in control but the $\bar{X}$ chart is out of control, the estimate of σ' which is obtained from the R chart will be a better estimate of the standard deviation of the underlying universe than the value obtained by calculating the "root mean square" deviation. For example:

The original data of Figure 122 were shown on page 14. $\bar{R}$ for samples of 5 was found to be 1.59. The d_2 factor for samples of 5 is 2.326.

$$\sigma' = \frac{\bar{R}}{d_2} = \frac{1.59}{2.326} = .68$$

This is a truer estimate of the standard deviation of the underlying process than the value of .86 which was calculated on page 130. This is because the distribution shifted its center during the period when the measurements were obtained, and the shift in center has inflated the estimate arrived at on page 130.

Range

The range is denoted by the symbol R. It is calculated as follows:

Let M = the largest value in a set of measurements
m = the smallest value
$R = M - m$

The range is used in quality control to detect certain types of assignable causes. Also, the average range of a series of samples which show control may be used as above to estimate σ'.

D-1.3 Shape

The third important characteristic of a distribution is its shape, or profile. Most distributions of actual observed data are irregular in shape, but sometimes distributions are found to be fairly uniform and symmetrical about the mean. Statistical techniques make use of a number of theoretical distribution shapes, which may or may not be approximated by the distributions observed in practice. Among the important theoretical shapes are the following:

(1) Normal distribution.
(2) Distributions which are symmetrical but not normal.
(3) Distributions showing various degrees and types of skewness.
(4) Distributions showing more than one mode or "peak."

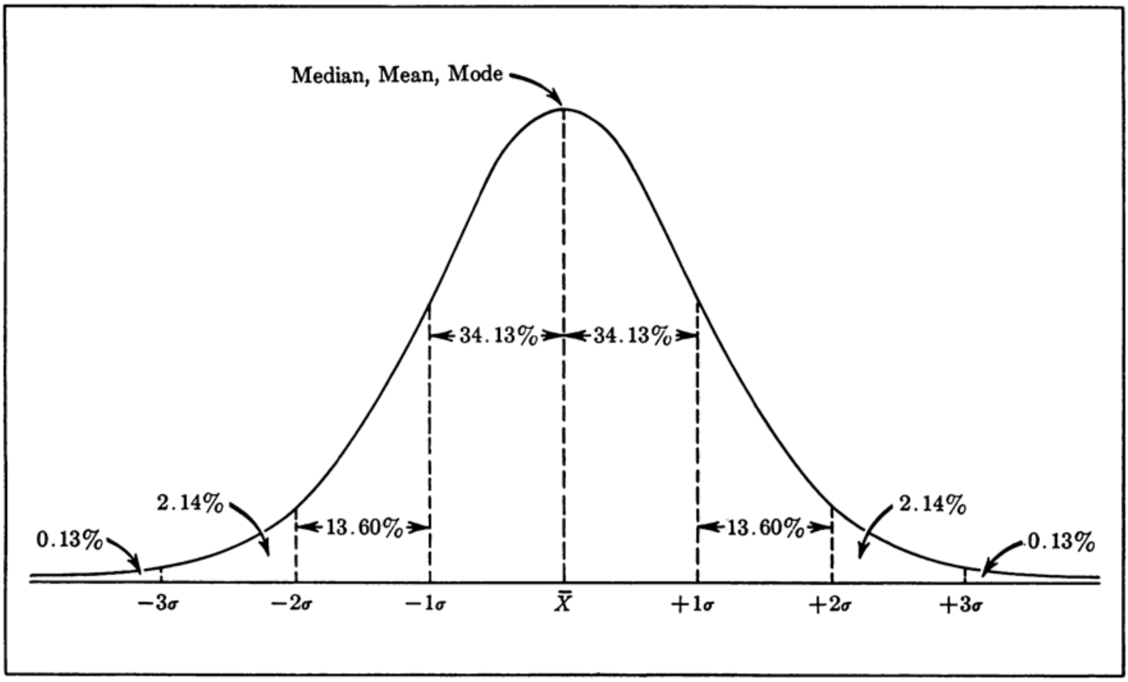


Fig. 124. Normal distribution.

D-1.4 Normal distribution

When statisticians speak of a "normal" distribution they mean one which is specifically defined by a certain mathematical equation. This distribution is perfectly symmetrical about its mean and has the familiar "bell shape" which is illustrated in Figure 124.

The equation for the Normal Distribution can be written in various ways, one of which is the following (Reference No. 6):

$$p(X) = \frac{1}{\sigma_X \sqrt{2\pi}} e^{-\frac{(X-\bar{X})^2}{2\sigma_X^2}}$$

where $p(X)$ is the ordinate to the curve for a given value of X (the measured variable).

This distribution has a number of important characteristics, among which are the following:

a. The areas on either side of the mean are equal.
b. About 68.26% of the total area is included within a distance of $\pm 1\ \sigma$ from the mean.
c. About 95.45% of the total area is included within a distance of $\pm 2\ \sigma$ from the mean.
d. About 99.73% (or virtually all) of the area is included within a distance of $\pm 3\ \sigma$ from the mean.

The table on page 133 gives a more complete listing of the percentages or areas associated with the Normal Curve.

For estimating the *percentage outside of limits* when a distribution is normal, calculate "t" as shown in the table for either the maximum or minimum limit. The percentage is given opposite the value of "t" in the appropriate column, depending on whether the value of "t" is found to be negative or positive.

The normal distribution is important in quality control for two reasons:

(1) Many distributions of quality characteristics of a product are reasonably similar to the normal distribution. This makes it possible to use the normal distribution for estimating percentages of product that are likely to fall within certain limits.

(2) Even when the distribution of product is quite far from normal, many distributions of statistical quantities, such as averages, tend to distribute themselves in accordance with the Normal Curve. For

TABLE I
NORMAL DISTRIBUTION

Percentage Outside of Max. $t = \frac{\bar{X} - \text{Max.}}{\sigma'}$			Percentage Outside of Min. $t = \frac{\text{Min.} - \bar{X}}{\sigma'}$		
t	If t is negative	If t is positive	t	If t is negative	If t is positive
0.0	50.0%	50.0%	0.0	50.0%	50.0%
0.1	46.0%	54.0%	0.1	46.0%	54.0%
0.2	42.1%	57.9%	0.2	42.1%	57.9%
0.3	38.2%	61.8%	0.3	38.2%	61.8%
0.4	34.5%	65.5%	0.4	34.5%	65.5%
0.5	30.8%	69.2%	9.5	30.8%	69.2%
0.6	27.4%	72.6%	0.6	27.4%	72.6%
0.7	24.2%	75.8%	0.7	24.2%	75.8%
0.8	21.2%	78.8%	0.8	21.2%	78.8%
0.9	18.4%	81.6%	0.9	18.4%	81.6%
1.0	15.9%	84.1%	1.0	15.9%	84.1%
1.1	13.6%	86.4%	1.1	13.6%	86.4%
1.2	11.5%	88.5%	1.2	11.5%	88.5%
1.3	9.7%	90.3%	1.3	9.7%	90.3%
1.4	8.1%	91.9%	1.4	8.1%	91.9%
1.5	6.7%	93.3%	1.5	6.7%	93.3%
1.6	5.5%	94.5%	1.6	5.5%	94.5%
1.7	4.5%	95.5%	1.7	4.5%	95.5%
1.8	3.6%	96.4%	1.8	3.6%	96.4%
1.9	2.9%	97.1%	1.9	2.9%	97.1%
2.0	2.3%	97.7%	2.0	2.3%	97.7%
2.1	1.8%	98.2%	2.1	1.8%	98.2%
2.2	1.4%	98.6%	2.2	1.4%	98.6%
2.3	1.1%	98.9%	2.3	1.1%	98.9%
2.4	0.8%	99.2%	2.4	0.8%	99.2%
2.5	0.6%	99.4%	2.5	0.6%	99.4%
2.6	0.5%	99.5%	2.6	0.5%	99.5%
2.7	0.4%	99.6%	2.7	0.4%	99.6%
2.8	0.3%	99.7%	2.8	0.3%	99.7%
2.9	0.2%	99.8%	2.9	0.2%	99.8%
3.0	0.1%	99.9%	3.0	0.1%	99.9%
3.1	0.1%	99.9%	3.1	0.1%	99.9%
3.2	0.1%	99.9%	3.2	0.1%	99.9%
3.3	0.05%	99.95%	3.3	0.05%	99.95%
3.4	0.03%	99.97%	3.4	0.03%	99.97%
3.5	0.02%	99.98%	3.5	0.02%	99.98%
3.6	0.02%	99.98%	3.6	0.02%	99.98%
3.7	0.01%	99.99%	3.7	0.01%	99.99%
3.8	0.01%	99.99%	3.8	0.01%	99.99%
3.9	0.01%	99.99%	3.9	0.01%	99.99%
4.0	0.00%	100.0 %	4.0	0.00%	100.0 %

this reason the normal distribution has important uses in statistical theory, including some of the theory which underlies control charts.

Tests for normality

The engineer may occasionally wish to test a set of data for normality—that is, to test whether it might reasonably have come from a normal population. Some of the difficulties of doing this are discussed on pages 78–79. Satisfactory tests for normality require fairly large amounts of data.

Three common methods of testing for normality are the following:

(1) Chi-square test. See Reference No. 13.

(2) Normal probability paper. The data

are plotted cumulatively on paper having special graduations. If the distribution is perfectly normal, the graph will be a straight line. See Reference No. 13.

(3) Calculation of skewness (lack of symmetry) and kurtosis (degree of flatness). These measures of non-normality may be tested for significance like any other statistical measures. See Reference No. 13.

The engineer should remember that for many quality control purposes it is not necessary to know whether a distribution is normal.

D-1.5 Distributions which are symmetrical but not normal

The engineer should not assume that all symmetrical distributions are normal. Examples of non-normal symmetrical distributions are shown in Figure 125.

The engineer will note that the areas in different portions of these curves are very different from the "normal" areas. Distribution A, which is flatter than the normal curve, is called a "platykurtic" distribution. Distribution B, which is more peaked than the normal curve, is called a "leptokurtic" distribution. The amount of flatness (or kurtosis) can be measured, if desired, by a measure known as "β_2." See Reference No. 19.

It is rarely necessary to measure flatness in quality control applications, but the engineer should be aware that such a characteristic exists.

D-1.6 Skewed distributions

Distributions are said to have *positive* or *negative* skewness depending on the direction of the longer tail. A distribution is skewed

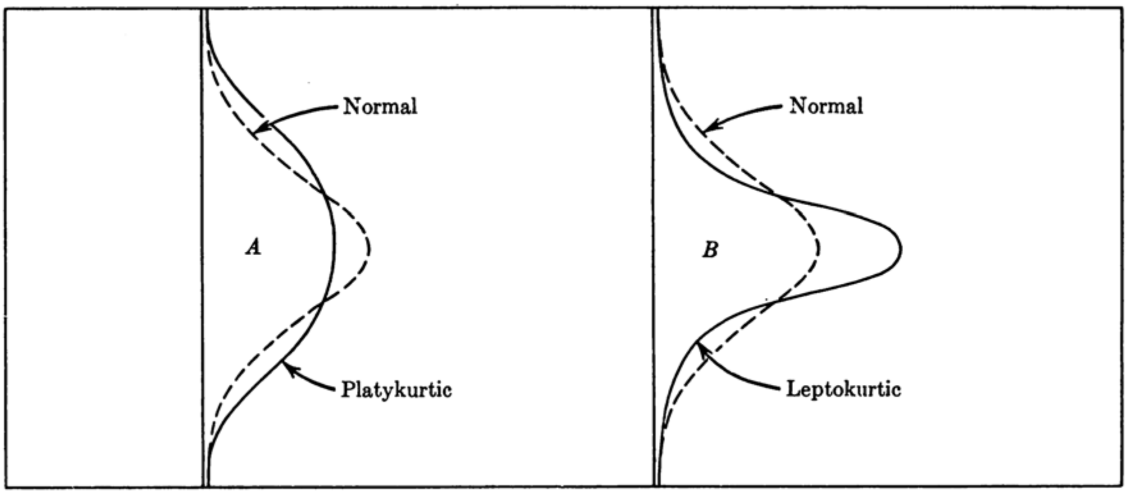


Fig. 125. Distributions which are symmetrical but not normal.

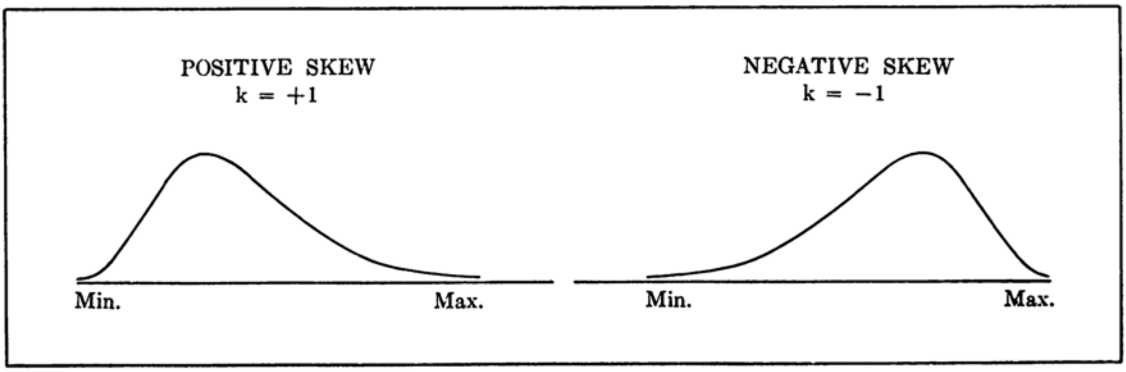


Fig. 126. Opposite types of skewness.

positively if the long tail is on the maximum side and negatively if the long tail is on the minimum side. These two types of skewness are illustrated in Figure 126.

The degree of skewness is measured by a factor called $\sqrt{\beta_1}$ or "k." See Reference No. 19.

One of the common theoretical distributions involving skewness is known as the "Second Approximation to the Normal Curve." Its equation is

$$p(X) = \frac{1}{\sigma_X \sqrt{2\pi}} e^{-\frac{(X-\bar{X})^2}{2\sigma_X^2}} \left[1 - \frac{k}{2}\left(\frac{X-\bar{X}}{\sigma} - \frac{(X-\bar{X})^3}{3\sigma^3}\right)\right]$$

using the same notation that was employed on page 132.

The distributions shown in Figure 126 are "Second Approximation" curves with $k = +1$

TABLE II
SECOND APPROXIMATION WITH $k = +1$

Percentage Outside of Max. $t = \frac{\bar{\bar{X}} - \text{Max.}}{\sigma'}$			Percentage Outside of Min. $t = \frac{\text{Min.} - \bar{\bar{X}}}{\sigma'}$		
t	If t is negative	If t is positive	t	If t is negative	If t is positive
0.0	43.3%	43.3%	0.0	56.7%	56.7%
0.1	39.4%	47.4%	0.1	52.6%	60.6%
0.2	35.8%	51.6%	0.2	48.4%	64.2%
0.3	32.5%	55.0%	0.3	44.6%	67.5%
0.4	29.3%	60.4%	0.4	39.6%	70.7%
0.5	26.4%	64.7%	0.5	35.3%	73.6%
0.6	23.8%	69.0%	0.6	31.0%	76.2%
0.7	21.5%	73.1%	0.7	26.9%	78.5%
0.8	19.4%	77.0%	0.8	23.0%	80.6%
0.9	17.4%	80.7%	0.9	19.3%	82.6%
1.0	15.8%	84.1%	1.0	15.9%	84.2%
1.1	14.3%	87.2%	1.1	12.8%	85.7%
1.2	12.9%	89.9%	1.2	10.1%	87.1%
1.3	11.6%	92.3%	1.3	7.7%	88.4%
1.4	10.4%	95.4%	1.4	5.6%	89.6%
1.5	9.3%	96.0%	1.5	4.0%	90.7%
1.6	8.3%	97.1%	1.6	2.9%	91.7%
1.7	7.4%	98.5%	1.7	1.5%	92.6%
1.8	6.5%	99.3%	1.8	0.7%	93.5%
1.9	5.7%	99.9%	1.9	0.5%	94.3%
2.0	4.9%	100.0 %	2.0	0.0%	95.1%
2.1	4.2%	—	2.1	—	95.8%
2.2	3.6%	—	2.2	—	96.4%
2.3	3.1%	—	2.3	—	96.9%
2.4	2.6%	—	2.4	—	97.4%
2.5	2.1%	—	2.5	—	97.9%
2.6	1.7%	—	2.6	—	98.3%
2.7	1.4%	—	2.7	—	98.6%
2.8	1.1%	—	2.8	—	98.9%
2.9	0.9%	—	2.9	—	99.1%
3.0	0.7%	—	3.0	—	99.3%
3.1	0.5%	—	3.1	—	99.5%
3.2	0.4%	—	3.2	—	99.6%
3.3	0.3%	—	3.3	—	99.7%
3.4	0.2%	—	3.4	—	99.8%
3.5	0.2%	—	3.5	—	99.8%
3.6	0.1%	—	3.6	—	99.9%
3.7	0.07%	—	3.7	—	99.9%
3.8	0.03%	—	3.8	—	99.9%
3.9	0.01%	—	3.9	—	99.99%
4.0	0.00%	—	4.0	—	99.99%

and $k = -1$ respectively. The tables on pages 135 and 136 show the percentages or areas associated with these two curves. The engineer should compare the percentages with those of the Normal Distribution on page 133.

As in the case of a normal distribution, these tables can be used for estimating the *percentage outside of limits*. Follow the rules on page 132 in calculating the value of "t" and read the percentage in the appropriate column.

Some distributions having higher degrees of skewness are shown on pages 57, 58 and 163.

D-1.7 Distributions having more than one mode or peak

Common distributions of this type are shown on pages 155, 158, 162, 166, 173, 174, 176 and 179. Bimodal or multi-modal distributions

TABLE III
SECOND APPROXIMATION WITH $k = -1$

	Percentage Outside of Max. $t = \frac{\bar{\bar{X}} - \text{Max.}}{\sigma'}$			Percentage Outside of Min. $t = \frac{\text{Min.} - \bar{\bar{X}}}{\sigma'}$	
t	If t is negative	If t is positive	t	If t is negative	If t is positive
0.0	56.7%	56.7%	0.0	43.3%	43.3%
0.1	52.6%	60.6%	0.1	39.4%	47.4%
0.2	48.4%	64.2%	0.2	35.8%	51.6%
0.3	44.6%	67.5%	0.3	32.5%	55.0%
0.4	39.6%	70.7%	0.4	29.3%	60.4%
0.5	35.3%	73.6%	0.5	26.4%	64.7%
0.6	31.0%	76.2%	0.6	23.8%	69.0%
0.7	26.9%	78.5%	0.7	21.5%	73.1%
0.8	23.0%	80.6%	0.8	19.4%	77.0%
0.9	19.3%	82.6%	0.9	17.4%	80.7%
1.0	15.9%	84.2%	1.0	15.8%	84.1%
1.1	12.8%	85.7%	1.1	14.3%	87.2%
1.2	10.1%	87.1%	1.2	12.9%	89.9%
1.3	7.7%	88.4%	1.3	11.6%	92.3%
1.4	5.6%	89.6%	1.4	10.4%	95.4%
1.5	4.0%	90.7%	1.5	9.3%	96.0%
1.6	2.9%	91.7%	1.6	8.3%	97.1%
1.7	1.5%	92.6%	1.7	7.4%	98.5%
1.8	0.7%	93.5%	1.8	6.5%	99.3%
1.9	0.5%	94.3%	1.9	5.7%	99.9%
2.0	0.0%	95.1%	2.0	4.9%	100.0%
2.1	—	95.8%	2.1	4.2%	—
2.2	—	96.4%	2.2	3.6%	—
2.3	—	96.9%	2.3	3.1%	—
2.4	—	97.4%	2.4	2.6%	—
2.5	—	97.9%	2.5	2.1%	—
2.6	—	98.3%	2.6	1.7%	—
2.7	—	98.6%	2.7	1.4%	—
2.8	—	98.9%	2.8	1.1%	—
2.9	—	99.1%	2.9	0.9%	—
3.0	—	99.3%	3.0	0.7%	—
3.1	—	99.5%	3.1	0.5%	—
3.2	—	99.6%	3.2	0.4%	—
3.3	—	99.7%	3.3	0.3%	—
3.4	—	99.8%	3.4	0.2%	—
3.5	—	99.9%	3.5	0.2%	—
3.6	—	99.9%	3.6	0.1%	—
3.7	—	99.9%	3.7	0.07%	—
3.8	—	99.9%	3.8	0.03%	—
3.9	—	99.99%	3.9	0.01%	—
4.0	—	99.99%	4.0	0.00%	—

usually result from the presence of more than one system of causes.

D-2 DISTRIBUTIONS DERIVED FROM SAMPLES

D-2.1 Sampling distributions

In general, the most convenient and useful way to collect data is in small groups called *samples.* For example we might measure 5 pieces of product occasionally and record them as in Figure 127. The x's represent the individual pieces of product and the large dot is their average.

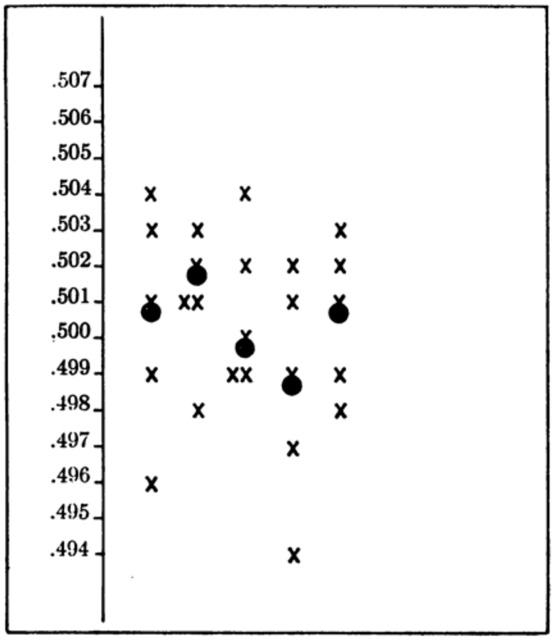


Fig. 127. Samples of 5 and their averages.

We could group the individual measurements together, if we wished, to form a frequency distribution.

In addition to the individuals, however, we now have a number of *averages.* The averages do not spread as widely as the individual measurements.

If we had enough averages and grouped them together, we would find that they tended to form a frequency distribution of their own, which would be considerably narrower than the distribution of individuals. This can be seen in Figure 128.

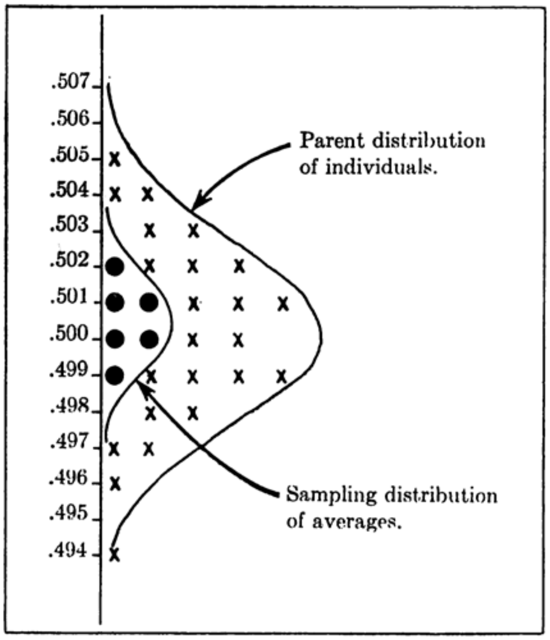


Fig. 128. Frequency distribution of data similar to Figure 127. A smooth curve has been drawn around the x's and also around the dots.

The distribution of sample averages is called a *sampling distribution* since, in order to obtain it, we must have a series of samples. In the same way, a series of ranges calculated from samples will form a sampling distribution of ranges. A series of percentages calculated from samples will form a sampling distribution of percentages. A series of counts obtained from samples will form a sampling distribution of counts.

Sampling distributions form the basis of most control charts. The sampling distributions mentioned above form the basis of $\bar{X}$ and R charts, p charts and c charts respectively.

D-2.2 Parent distribution

The underlying distribution of the process from which the samples were taken is referred to as the *parent distribution* in order to distinguish it from the distributions derived from samples. Other names for the parent distribution are:

(1) Universe.

(2) Population.

(3) Distribution of individuals.

D-2.3 Relationships between parent and sampling distributions

Sampling distributions are related mathematically to the parent distribution from which the samples came. The parent distribution determines (a) the standard deviation of the sampling distribution, (b) its center or average, and (c) to a certain extent its shape.

Some of the relationships between parent and sampling distributions are rather involved, but in the case of the *sampling distribution of averages* the relationships are quite simple. For this reason, the sampling distribution of averages has been used in the illustrations which follow.

The sampling distribution of averages has the following relation to the parent:

(1) The center of the sampling distribution of averages is the same as that of the parent.

(2) The shape of the sampling distribution of averages is governed to some extent by the parent, but in general the sampling distribution tends to follow the normal curve quite closely, even when the parent distribution is irregular, skewed, triangular or square. For most practical purposes in engineering work, it can be assumed that the sampling distribution of averages is approximately normal.

The engineer should note, however, that this would not hold for parent distributions having very extreme forms (such as U-shapes or J-shapes), unless the samples are very large. If the engineer should encounter such a case in practice, he may expect to find the shape of the sampling distribution significantly affected by the parent.

(3) The standard deviation of the distribution of sample averages is related to the standard deviation of the parent distribution as follows:

$$\text{Standard deviation of averages} = \frac{\text{standard deviation of parent}}{\sqrt{n}}$$

where n = number of individuals in each sample average.

If $n = 5$, the standard deviation of averages $= \frac{1}{\sqrt{5}}$ times the standard deviation of the parent. This is $\frac{1}{2.236}$, or approximately 45%.

The control limits which appear on an $\bar{X}$ chart are merely the 3 sigma limits for the sampling distribution of averages.

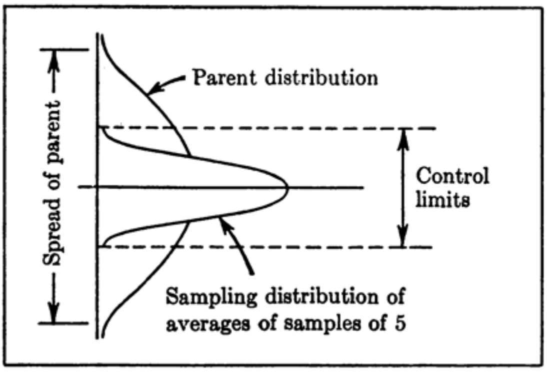


Fig. 129. Control limits for averages: Samples of 5.

In the drawing above, the area of the sampling distribution is shown as approximately equal to that of the parent.

D-3 METHODS OF PLOTTING A FREQUENCY DISTRIBUTION

Consider the measurements shown on page 14 (Gain in Db.). There are 100 measurements in all and they range from 8.3 to 12.8. This is a spread of 4.5 db. When a frequency distribution is to be made from a large number of observations scattered over many different values on the scale of measurement, it is usually convenient to group the data into intervals or cells. Arrange the intervals in such a way that there will be, if possible, from 10 to 30 cells. In the present example it would be convenient to divide the spread into cells of 0.2 db. each. Tally the number of observations that fall into each cell as shown in Figure 130.

The cell boundaries should be arranged in such a way that there is only one cell in which

	Gain in Db.
12.7–12.8	x
12.5–12.6	
12.3–12.4	xxx
12.1–12.2	x
11.9–12.0	xxx
11.7–11.8	xx
11.5–11.6	xxxx
11.3–11.4	xxxxxxxxxx
11.1–11.2	xxxxxxxxxx
10.9–11.0	xxxxxxxx
10.7–10.8	xxxxxxxxxxx
10.5–10.6	xxxxxxx
10.3–10.4	xxxx
10.1–10.2	xxxxxxxxxxxxxxx
9.9–10.0	xxxxxx
9.7–9.8	xxxxxx
9.5–9.6	xx
9.3–9.4	xxx
9.1–9.2	
8.9–9.0	
8.7–8.8	
8.5–8.6	x
8.3–8.4	xxx
8.1–8.2	

Fig. 130. Distribution of measurements in Figure 13 (page 14).

a given measurement may be placed. The width of all cells should be the same, and the total number of observations in all cells should not be less than 25.

Histograms

A more formal way of plotting a frequency distribution of observed values is to erect a series of columns, each having a width equal to the cell width. The height of the column represents the number of observations in each cell. Such a representation of data is called a histogram. It is used in the same way as any frequency distribution. See Figure 131 on page 140.

Other methods of plotting a frequency distribution are given in Reference No. 2.

Fitting a curve to data

Engineers are frequently called on to "fit" a theoretical curve to a set of observed data. The most common example is that of comparing actual data with a theoretical Normal Curve. In order to draw a theoretical curve, the engineer must first calculate the average and standard deviation of the observed data. Second, he must know the ordinates or areas of the theoretical curve he wishes to reproduce (usually obtainable from tables). Third, he must be able to adjust the plotting scales for the observed data and the theoretical curve so that the areas will be equal.

A simple way to do this is as follows:

(1) Divide the data into a convenient number of cells and compute the average and standard deviation as shown on page 130. Call these $\bar{X}'$ and σ' respectively.

(2) Translate the cell boundaries into ± values of sigma, or standard deviations from the average. Thus, if B = the cell boundary in terms of absolute units, take $\frac{B - \bar{X}'}{\sigma'}$ to obtain the cell boundary in terms of sigma.

(3) Look up each pair of cell boundaries in the Table of Percentages or Areas for the theoretical curve you have in mind. The difference between the two percentages (one for each boundary) gives the theoretical percentage that should fall in each cell.

(4) Mark off the cells at the bottom of a piece of graph paper, indicating both absolute units and ± values of sigma. Plot a point at the midpoint of each cell corresponding to the theoretical percentages. Choose any convenient vertical scale.

Draw a smooth curve through the points representing the theoretical distribution.

(5) Taking the observed data which are to be used for comparison, convert the frequencies for each cell into "percentage of total frequencies" by taking $\frac{\text{Observed}}{\text{Total}}$. Erect bars for each cell corresponding to the observed percentages. Use the same vertical scale as for the theoretical distribution.

(6) You now have a drawing with (a) a smooth curve for the theoretical distribution and (b) a histogram for the observed data. (See Figure 132 on page 141.) The

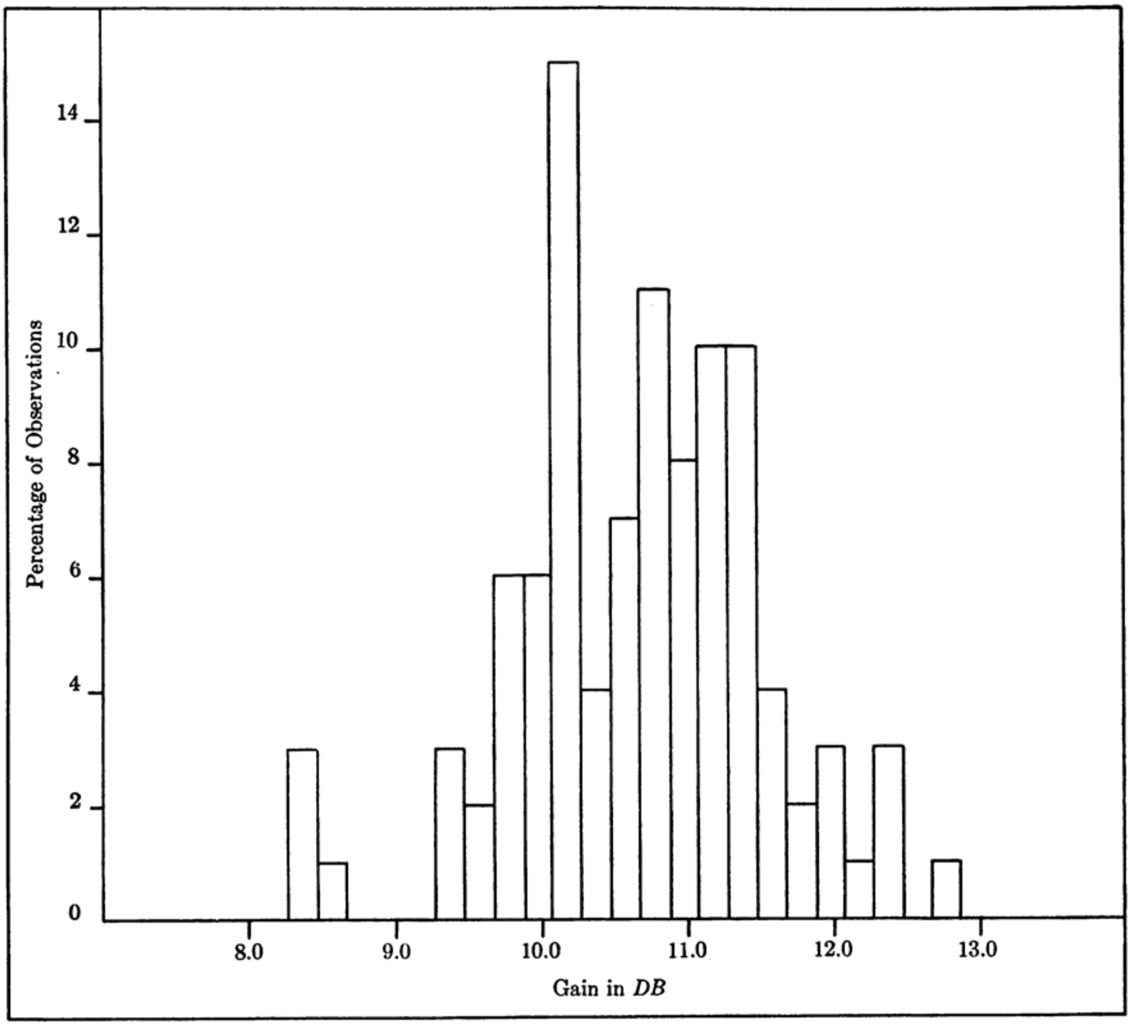


Fig. 131. Histogram of the data in Figure 130.

distribution and the histogram have the same average and standard deviation, and the same general area.

For presentation it is generally desirable to change the scale markings at the bottom of the drawing to show convenient absolute values: for example, the values corresponding to the midpoints of the cells instead of their boundaries, or any other convenient values.

A chi-square test can be used, if desired, to obtain a numerical measure of the "goodness of fit." See References No. 5 and 13.

D-4 PRACTICAL USES OF FREQUENCY DISTRIBUTIONS

A frequency distribution of individual measurements found in a sample is likely to exhibit some of the characteristics of the parent distribution of product. Such observed distributions are useful for

(1) Comparing a collection of units with the specification.

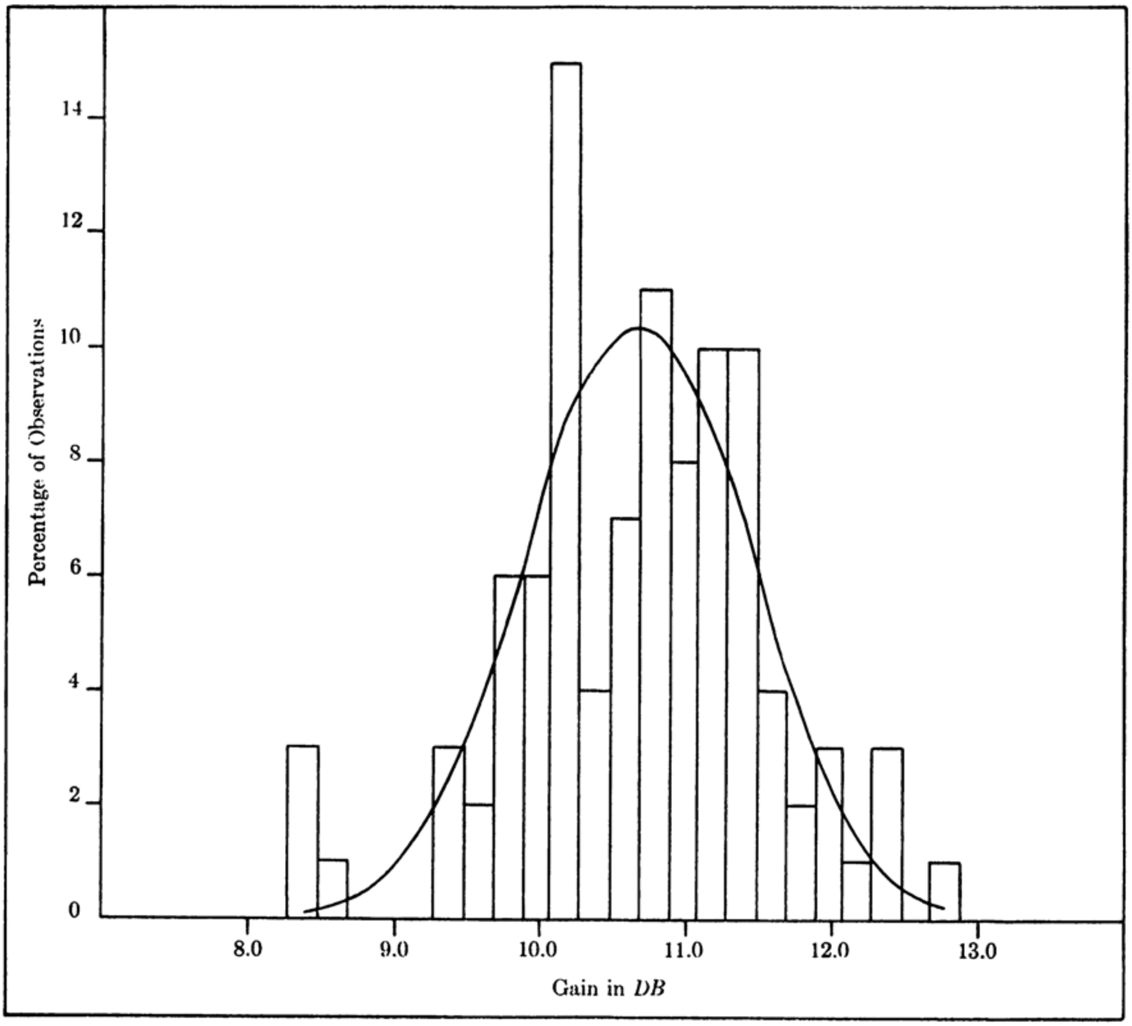


Fig. 132. Theoretical Normal Curve compared with the actual data in Figure 131.

(2) Suggesting the shape of the parent distribution or universe.

(3) Indicating certain discrepancies or peculiarities in the data, such as coarseness, gaps or screening.

Observed distributions should, however, be used with caution, and the engineer should beware of attempting to get more information out of them than a distribution can give. Do not attempt to draw general conclusions from distributions unless the data are in control. This can be determined only by plotting the data on control charts.

PART E
Correlation

E-1 GRAPHICAL METHODS OF STUDYING CORRELATION

E-1.1 Scatter diagrams

The simplest way to study correlation is to plot a scatter diagram. Obtain values for the two variables, x and y, in pairs. That is, measure x on a certain unit and y on the same unit, identifying them as a pair. One point on the scatter diagram represents one pair of x and y values. Figures 133–135 show typical scatter diagrams which indicate (a) positive correlation (b) negative correlation and (c) no correlation.

Correlation is said to be positive if the y values increase as the x values increase. Correlation is said to be negative if the y values decrease as the x values increase. There is absence of correlation if the y values may be either higher or lower as the x values increase.

If the scale markings on Figure 133 and Figure 134 are spaced alike, Figure 133 shows a higher degree of correlation than Figure 134. The more the points scatter, the less is the correlation.

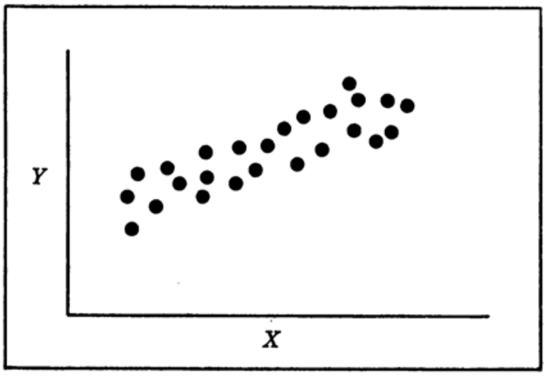


Fig. 133. Scatter diagram: Positive correlation.

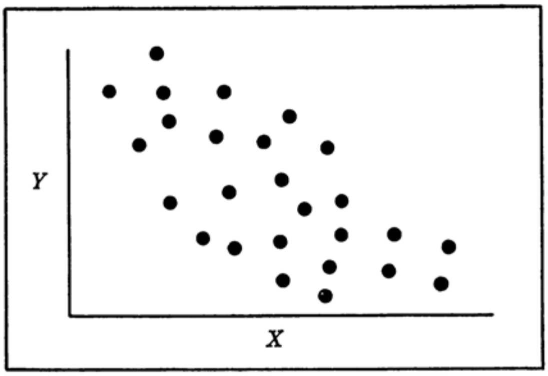


Fig. 134. Scatter diagram: Negative correlation.

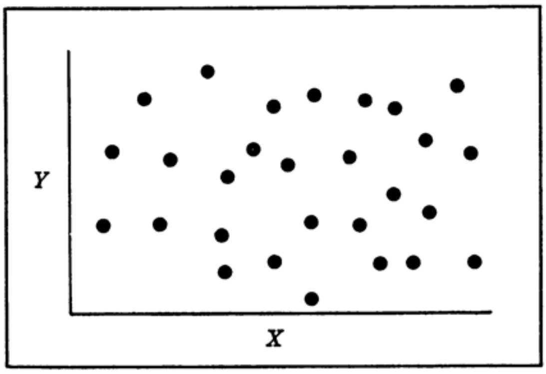


Fig. 135. Scatter diagram: No correlation.

E-1.2 Trend arrangements

Since correlation can be defined as a trend in y with increasing (or decreasing) values of x, it is possible to use a control chart to test for correlation. This technique is similar to the scatter diagram except that it is possible to apply a control chart test to see whether a trend really exists. Proceed as follows:

(1) Arrange the pairs of measurements in ascending order of x. Then, ignoring x, divide the data into convenient subgroups and plot a standard control chart for y. This method can be followed with either variables or attributes data.

(2) If the control chart shows an upward

trend, there is positive correlation. If the trend is downward, there is negative correlation. If the chart stays in control, there is no reason to conclude that the two variables are correlated.

The control chart will also show whether there is a change in the variability of y as x increases. There may be a change in variability when there is no change in the average.

In addition, the control chart may indicate that some of the data are "wild" or out of control. It is necessary to make a special allowance for such data in any estimates involving correlation.

For maximum sensitivity use an $\bar{X}$ and R chart, and collect the data in such a manner that there will be 2, 3, 4 or 5 measurements of y at each of several values of x. The subgroups will then be rational subgroups with respect to variations in x.

Instead of using subgroups it is possible to

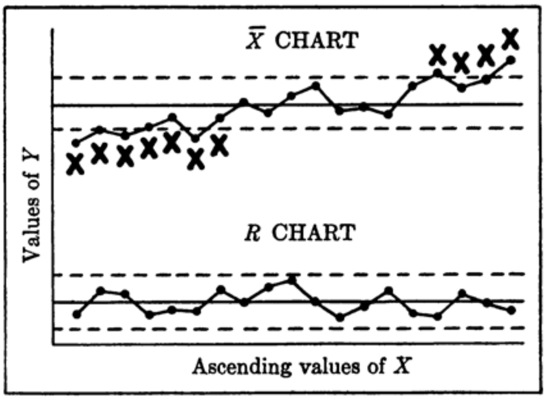


Fig. 136. Trend arrangement: Positive correlation between x and y.

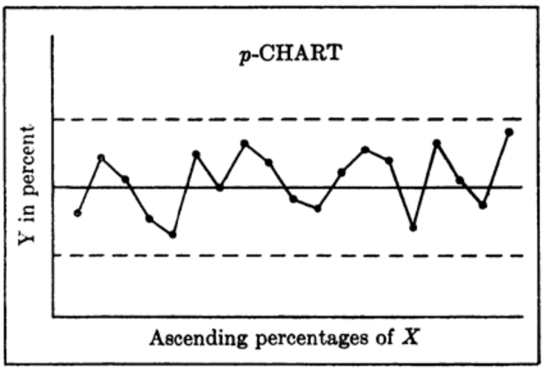


Fig. 137. Trend arrangement: No correlation between x and y.

plot the individual values of y, basing the control limits on the moving range. This type of chart is not as sensitive as an $\bar{X}$ and R chart.

E-1.3 Determining the slope of the correlation

In many cases it is possible to tell, without calculation, the approximate "trend line" which the data appear to follow. Frequently, however, the engineer wishes to draw a line which will represent the relationship as exactly as possible. If he attempts to do this by eye, he may or may not be successful. The degree of success depends largely on how far the data scatter. See Figures 138 and 139.

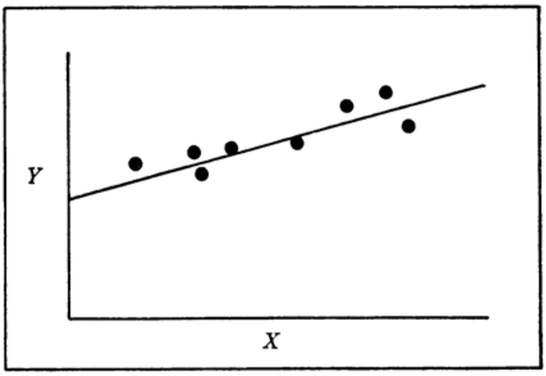


Fig. 138. Few engineers would disagree on this.

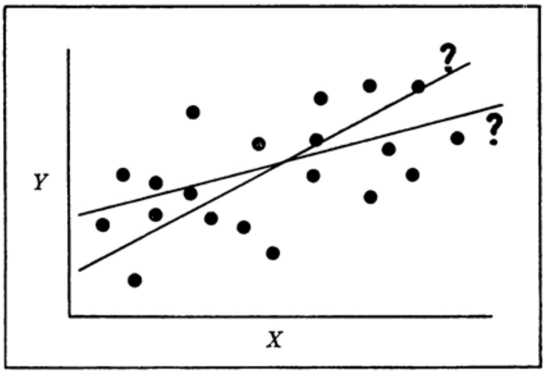


Fig. 139. There may be a difference of opinion on this.

In doubtful cases the engineer should calculate the "regression line of y on x" (or of x on y) as explained below.

E-2 REGRESSION LINES

A "regression line" or "line of regression" is a line which represents the slope of correlated data as accurately as possible. The "line of best fit" is defined as the line which will make the *sum of the squares* of the deviations from the line a minimum. The method of calculating this line is known as the "method of least squares."

E-2.1 How to calculate the line of regression

Let x = the value of one variable.
y = the value of the other variable.

Draw up a table as shown in Figure 140.

Variables	x	y	xy	x^2
Data				
Totals				
Symbols for Totals	Σx	Σy	Σxy	Σx^2

Fig. 140. Table for calculating a line of regression.

The equation for the straight line regression is

$$y = mx + c$$

where

$$m = \frac{\Sigma xy - \dfrac{(\Sigma x)(\Sigma y)}{n}}{\Sigma x^2 - \dfrac{(\Sigma x)^2}{n}}$$

$$c = \frac{(\Sigma x)(\Sigma xy) - (\Sigma y)(\Sigma x^2)}{(\Sigma x)^2 - n(\Sigma x^2)}$$

n = the number of pairs of x, y values.

Substitute these values into the equation and plot the sloping line. As a check on your calculations, make sure the line passes through the point $\bar{x}$, $\bar{y}$.

Alternative method

If the engineer has already calculated the coefficient of correlation, "r," as shown on page 146, this can be used to calculate the line of regression. The method is shown in Figure 143.

E-2.2 Regression of y on x and x on y

The line of regression described in paragraph E-2.1 is known as the "regression of y on x." This is used to predict or estimate y values when given x. It assumes that x is the "independent variable" whose values can be fixed or which is accurately known. It assumes that y is a "dependent variable" whose values will change with any change in the value of x.

It would also be possible to calculate a line for the "regression of x on y." In that case we would assume that y is the independent variable while x is dependent, and we would predict or estimate x values when given y. To calculate this line, merely reverse x and y in the equations given above.

The two lines of regression, y on x and x on y, will not ordinarily be the same for any actual set of data. The engineer must decide which variable to consider as independent and determine the line accordingly.

If the engineer has no reason for considering either variable to be independent, he may wish to calculate both lines of regression and compare them. For many practical purposes the most useful line will be a line midway between the line of regression of x on y and the line of regression of y on x.

E-2.3 How to put control limits around a line of regression

To put control limits around a line of regression, proceed as follows:

(1) Calculate the coefficient of correlation, "r," as shown in Sub-section E-3.

(2) Calculate the "standard error of estimate," σ_e, as follows:

$$\sigma_e = \sigma_y \sqrt{1 - r^2}$$

(3) Use σ_e to establish regular 3 σ control limits about the regression line as in Figure 141 on page 146.

It is also possible to plot the successive *deviations* from the regression line, using control limits based on the moving range or an $\bar{X}$ and R chart. If the deviations are found to be out of control, we cannot be confident that the regression line really fits the data.

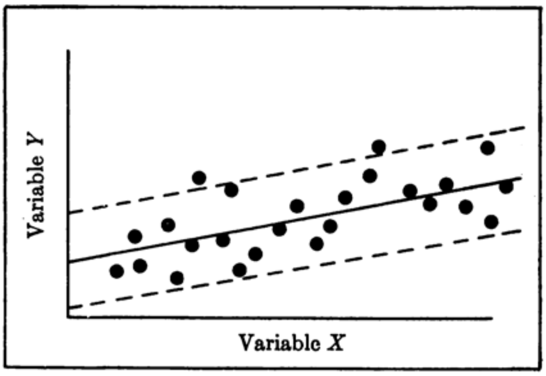


Fig. 141. Control limits around the line of regression.

E-3 FORMAL CORRELATION ANALYSIS

Whether or not he has calculated a line of regression as in Sub-section E-2, the engineer may wish to obtain a mathematical measure of the degree of correlation between two variables. The degree of correlation is measured by the "correlation coefficient" (designated by the symbol "r"). It is calculated as follows:

$$r = \frac{\frac{1}{n}\Sigma[(x - \bar{x})(y - \bar{y})]}{\sigma_x \sigma_y}$$

where x and y are the values of the two variables, respectively, and n is the number of pairs. The terms in the denominator (σ_x and σ_y) can be obtained by taking the "root mean square" deviation of all the values of the variable from their average as on page 130, or alternatively by filling in the form shown in Figure 142. Figure 143 shows how the correlation coefficient can be used to calculate lines of regression if the engineer has not previously done so.

General meaning of "r"

The value of "r" will be positive if there is positive correlation between the variables and negative if there is negative correlation. The general meaning of "r" is summarized in Figure 144.

Precautions

In using the coefficient of correlation it is necessary to observe the following rules and precautions:

(1) While "r" is a measure of the linear relationship between x and y as it exists in a given set of numbers, even a high value of r does not imply that x and y are related as cause and effect. It is possible to have a high degree of mathematical relationship with no causal relationship whatsoever. For example: statistics show that teachers' salaries and the national consumption of liquor tend to go up and down at the same time. This does not prove that when teachers get higher salaries they spend the additional money on liquor.

It is possible that two variables which are not related to each other may both be related to a third variable. This will cause the first two variables to show a mathematical relationship when there is no real cause and effect.

The engineer should make sure that there is an engineering reason to account for the correlation before he attempts to draw conclusions from a calculated value of r.

(2) While unrelated sets of numbers should have zero correlation *on the average*, individual samples may, as a result of sampling fluctuation, have values of "r" above or below zero. Consequently, the fact that "r" is other than zero does not necessarily indicate that two sets of numbers are related.

Do not draw any conclusions based on the correlation coefficient until you have tested it for significance as follows:

(a) Multiply r by $\sqrt{n}$, where n is the number of pairs of measurements used to determine r. Call this "t."

(b) If t is greater than 3, consider that the correlation is significant. If t is less than 3, the correlation may not be significant. The lower the value of t. the less likely it is that the correlatior is significant.

If the correlation is not definitely significant as determined above, this may be due to (a) real absence of correlation or (b) insufficient data. If the engineer has reason to believe that correlation should exist he may wish to obtain more data, calculate a new value of r, and test the new value for significance.

Cell width for X. / Cell width for Y.	47000 to 51490	51500 to 55990	56000 to 60490	60500 to 64990	65000 to 69490	69500 to 73990	74000 to 78490	78500 to 82990	y	f	fy	fy^2	Σfxy
64.0–68.4									+4	0	0	0	0
59.5–63.9								xxx	+3	3	+9	27	+27
55.0–59.4					x		xxx xxxx	xxxx	+2	12	+24	48	+52
50.5–54.9				x	xx		xxx	x	+1	7	+7	7	+8
46.0–50.4				xx	x				0	3	0	0	0
41.5–45.9		xx		x	x				−1	4	−4	4	+7
37.0–41.4	x	xx							−2	3	−6	12	+20
32.5–36.9	x								−3	1	−3	9	+12
28.0–32.4	x	x							−4	2	−8	32	+28
23.5–27.9									−5	0	0	0	0
x	−4	−3	−2	−1	0	+1	+2	+3	Totals	35	+19	139	+154
f	3	5	0	4	5	0	10	8	35	n	(A) +.543	(B) 3.971	(C) +4.400
fx	−12	−15	0	−4	0	0	+20	+24	+13	(D) +.371			
fx^2	48	45	0	4	0	0	40	72	209	(E) 5.971			

To obtain the values in the $\Sigma\, fxy$ column, calculate fxy separately for each one of the "boxes." Then total all the fxy's for each row and enter in the $\Sigma\, fxy$ column.

$\bar{x} = D = +.371$ $\qquad \bar{x}^2 = .138 = F$

$\bar{y} = A = +.543$ $\qquad \bar{y}^2 = .295 = G$

$\bar{x}\bar{y} = D \text{ times } A = .2015 = H$

$\sigma_x = \sqrt{E - F} = \sqrt{5.833} = 2.415 = J$

$\sigma_y = \sqrt{B - G} = \sqrt{3.676} = 1.917 = K$

$\sigma_x \sigma_y = J \text{ times } K = 4.630 = L$

$$\text{Coefficient of correlation} = \frac{C - H}{L} = \frac{+4.1985}{4.630} = +.907 = r$$

To test whether the apparent correlation is significant, make a t-test as follows:

$$t \text{ (no. of sigma)} = r \text{ times } \sqrt{n} = .907 \times \sqrt{35} = 5.38$$

If t is greater than 3 consider that there is significant correlation.

Fig. 142. Calculation of the coefficient of correlation.

$$
\begin{aligned}
&\text{Cell width for } X \text{ values} = 4500 = w \\
&\text{Cell width for } Y \text{ values} = 4.5 \quad = m \\
&\sigma_x = J \text{ times } w = 10868 \\
&\sigma_y = K \text{ times } m = 8.626 \\
&\bar{X} = \text{Midpoint of cell } x = 0, \text{ plus } (D \text{ times } w) \\
&\qquad 67245 + 1670 = 68915 \\
&\bar{Y} = \text{Midpoint of cell } y = 0, \text{ plus } (A \text{ times } m) \\
&\qquad 48.2 + 2.44 = 50.64
\end{aligned}
$$

Regression of Y on X:

$$b = \frac{r \text{ times } \sigma_y}{\sigma_x} = \frac{.907 \text{ times } 8.626}{10868} = .0007199$$

$$a = \bar{Y} - (b \text{ times } \bar{X}) = 50.64 - 49.61 = 1.03$$

$$Y = a + bX$$

Regression of X on Y:

$$b = \frac{r \text{ times } \sigma_x}{\sigma_x} = \frac{.907 \text{ times } 10868}{8.626} = 1142.7$$

$$a = \bar{X} - (b \text{ times } \bar{Y}) = 68915 - 57866 = 11049$$

$$X = a + bY$$

Fig. 143. Calculation of lines of regression. The values are obtained from Figure 142.

Relationship Between x and y		
$r = +1.0$	Strong, positive.	As x increases, y always increases.
$r = +0.5$	Weak, positive.	As x increases, y tends in general to increase.
$r = 0$	No correlation.	x and y are independent.
$r = -0.5$	Weak, negative.	As x increases, y tends in general to decrease.
$r = -1.0$	Strong, negative.	As x increases, y always decreases.

Fig. 144. Meaning of the coefficient of correlation.

E-4 OTHER INFORMATION ON CORRELATION

For further information on correlation see References No. 5, 13 and 26. These references give information on multiple correlation, partial correlation and curvilinear correlation, none of which are covered in this Handbook.

PART F
Control Chart Patterns

This part of the Handbook gathers together much of the information needed by the engineer in interpreting control chart patterns for process capability studies or designed experiments. It is assumed that the engineer is already familiar with the elementary theory of control charts as given on pages 5–12 and also with the method of testing control chart patterns for unnaturalness as given on pages 23–30. It is also assumed that the engineer is familiar with the practical analysis of shop charts as discussed in Part C of the Shop Section and of process capability studies as discussed in Engineering Part A. The present material does not duplicate either the elementary theory or the practical analyses. It is intended to be used as supplementary reference material for those interested in a more thorough understanding of control charts.

F-1 CONTROL CHART THEORY

F-1.1 Control charts in general

The control chart in essence is a set of statistical limits applied to a sequence of points representing a process under study. The data comprising each individual point are random, but the points themselves are plotted in some deliberately chosen non-random arrangement selected to represent the most important variable. See Figure 145.

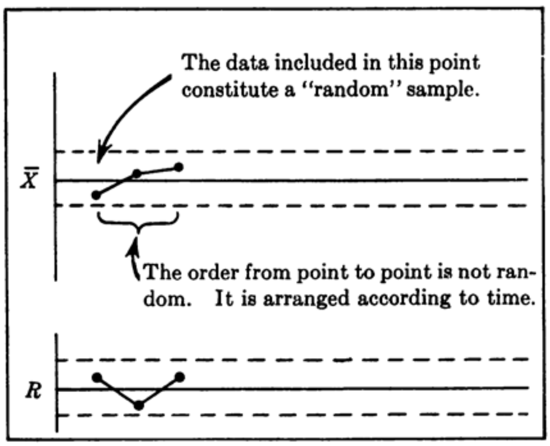


Fig. 145. Theory of the control chart: meaning of random samples.

In a process capability study, the most important variable is usually considered to be time. Consequently, the points are plotted in the order of time; that is, in the order of production or (if the test method is one of the factors to be studied) in the order of testing. Where a process capability study is to be broken down by "production paths," the separate paths constitute an important variable, and therefore the points are plotted according to these paths. In a designed experiment the important variables are in turn the different factors included in the experiment. Consequently, the points are arranged and re-arranged according to these factors.

The control chart tests the arbitrary or non-random arrangements of points to determine whether they behave as if they were random. If the plotted points indicate nothing but randomness, this tends to show that the variable which formed the base of the arrangement is not a significant variable.

On the other hand, if the points indicate that non-randomness has entered the data, this tends to show that the variable on which the arrangement was based is actually a significant variable.

F-1.2 Assignable causes

The control chart has a unique ability to detect and identify causes. First the pattern is tested for evidence of unnaturalness as explained on pages 23–30. Unnatural patterns are then associated with appropriate causes. The "causes" are extraneous disturbances or influences which interfere with or change the

Assignable Causes Associated with things which are:	*Non-Assignable Causes* Associated with things which are:
Unnatural	Normal
Disturbed	Natural
Unstable	Stable
Non-homogeneous	Undisturbed
Mixed	Homogeneous
Erratic	Coming from a single distribution
Abnormal	Not changing
Shifting	Steady
Unpredictable	Predictable
Inconsistent	Same
Out-of-the ordinary	Consistent
Different	Statistically constant
Important	Non-significant
Significant	

Fig. 146. Conditions associated with assignable and non-assignable causes.

ordinary behavior of the process.

The causes which disturb a process are called "assignable causes," because the reasons for them can be identified or "assigned." Assignable causes are always associated with "unnatural" behavior—that is, with something out of the ordinary or some change in the cause system.

The "natural" variation in the process is also the result of causes, but these are known as "non-assignable" causes. Non-assignable causes are relatively small in magnitude. They are also numerous, closely intermingled and statistically in balance. It is not possible to identify or "assign" such causes without going to special effort.

The following is a description of how the control chart detects assignable causes.

(1) When we break a series of measurements into very small samples and plot them, (for example, in the order of time), this forces any disturbing causes to show up in one of two ways.

(a) *Some disturbances come and go in the process periodically (occasional disturbances).*

A machine setter changing a setting will cause a periodic disturbance in the process. Occasional causes like this will not affect observations that are close together, as in a small sample, but they will affect observations that are farther apart or in different samples.

These are called causes operating "between samples."

(b) *Other disturbances do not come and go on an occasional basis but exist in the process for considerable periods (continuous disturbances).*

For example, a regular mixture of product made by several machines or operators may constitute such a disturbance. Continuous causes like this will affect the observations in a single small group or sample as much as they affect observations that are farther apart or in different samples.

These are called causes operating "within samples."

(2) "Between sample" causes tend to produce the effects known as instability, cycles, trends, systematic variation etc. "Within sample" causes tend to produce the effects known as stratification and stable forms of mixture. "Between sample" causes tend to have patterns too wide for the control limits, while "within sample" causes tend to have patterns too narrow for the control limits. The only thing that will produce a long-continued natural pattern on the control chart is the absence of disturbing causes within or between samples.

Not all assignable causes in a process are bad or need to be eliminated. For example, toolwear is an assignable cause, but it is accepted as an essential part of any process which involves tooling. Assignable causes may also be the source of important information, as shown on pages 35 and 54.

As a rule, however, assignable causes need to be either eliminated or restricted in some way in the interests of economical manufacture. In any case, whether we intend to eliminate them or not, we need to be aware of their presence in the process and of their magnitude.

F-1.3 Rational sub-groups

One of the chief sources of the power of the control chart is the manner of planning the samples before data are even collected. The samples are planned in such a way that, to the best of our knowledge, the units in any one sample should be a "rational subgroup." A rational subgroup is one which we believe, for rational or logical reasons, is as free as possible from assignable causes. That is, if we believe that different machine settings may have an effect on the characteristic being plotted, we see that all units in the sample come from the same setting. If we believe that different batches of material may have an effect, we see that all units in one sample come from the same batch, etc. A *series* of samples will then show the effect of *differences* in machine settings, batches etc.

We say that a rational subgroup is one that represents, as nearly as possible, a homogeneous set of conditions. In general, we know that manufacturing conditions tend to change from time to time as a result of variables of which we may not be aware. Consequently, to obtain subgroups which have the best chance of being rational, we attempt to include in one sample units made as nearly as possible at the same time.

A small group of consecutively produced units from a process is likely to be a "rational subgroup." That is, it is likely to be made up of a randomly produced set of units representing the immediate state of the process at the time the sample was selected.

It is possible that, in spite of our precautions, the subgroups we believed were rational may contain assignable causes. In that case the causes tend to show up as the "within sample" causes referred to in paragraph F-1.2. "Within sample" causes are harder to interpret than "between sample" causes. For this reason, the careful collection of data in rational subgroups will greatly simplify the use of the control charts.

F-1.4 Order of production (or testing)

When theory states that the samples for control charts should be taken, whenever possible, in the order of production, keep in mind that this means the order of production as related to a single system of causes. The principle of the rational subgroup (see above) is always assumed when we say "order of production."

If a process checker is taking samples from a machine with multiple spindles, or multiple positions or heads, then a series of consecutive units from the machine as a whole will not give him a sample in the "order of production" as the term is used here. If the machine has six heads, he should take every sixth unit in order to get a sample in the order of production from a single head, etc.

Order of production is important primarily because it aids in obtaining a rational subgroup.

F-1.5 Technical terms associated with control charts

Control limits

Control limits are mathematical or statistical limits used to interpret the pattern on a control chart. Unless otherwise noted, the control limits referred to in this Handbook are 3 sigma control limits. Control limits are derived from a knowledge of distribution theory and apply to the particular statistic ($\bar{X}$, R, p, individual measurement etc.) which is being plotted on the control chart. It is important not to confuse "control limits" with specification limits, or with the so-called "natural limits" of the process, which show the natural spread of individual units.

Natural process limits

Three sigma limits for the individual units produced by a process in control are sometimes called the "natural" limits of the process. The natural limits have no necessary connection with specification limits or any other arbitrary limits. Natural limits may be either broader or narrower than the specifications set by engineers.

"Natural limits" are the limits which the process is able to hold when operating normally under the influence of non-assignable chance causes. The term "natural tolerance" is sometimes used in place of "natural limits." See page 61.

Centerline

The centerline on a control chart is a line which passes through the center of a real or assumed set of fluctuating points from which the centerline was calculated. On any particular control chart the centerline may or may not pass through the points actually plotted. For example, the centerline may have been fixed by engineering decision, or it may have been obtained from a series of past data rather than the data currently plotted.

Level

The level on a control chart is a line which passes through the center of the series of points actually plotted. The line may be drawn on the chart or it may be imaginary. The level may or may not be the same as the centerline, since a level is always related to the actual plotted points while the centerline may have been obtained from some other source. It is possible for the same control chart to show more than one level in its patterns.

F-2 INTERPRETATION OF $\bar{X}$ CHARTS

The $\bar{X}$ chart shows where the process is centered. It represents the average of the distribution which the process is creating. If the center of the distribution shifts, the $\bar{X}$ pattern will shift with it. If the center of the distribution follows a trend up or down, the $\bar{X}$ pattern will follow the same trend. The conditions which the $\bar{X}$ chart is intended to reflect are shown in Figures 147 and 148.

F-2.1 Causes affecting the $\bar{X}$ chart

The most common causes which will affect an $\bar{X}$ chart are the following:

(1) *Direct or "true" $\bar{X}$ causes.* These are causes capable of affecting an $\bar{X}$ chart directly. All such causes have one element in common: that is, when they enter the process they are capable of affecting all the product at once or in the same general way. When the temperature changes in a plating bath, it affects all the parts being plated. When a decision is made to use a thicker stock, all the parts become thicker. This type of cause is able to shift the *center* of a distribution without affecting its spread. It is the most common type of cause which shows up on the $\bar{X}$ chart.

The $\bar{X}$ chart can be affected in this way by

Changes in:
- Material.
- Operator.
- Inspector.
- Machine setting.
- Plating current.
- Temperature.
- Strength of solution.
- Time in oven or tank.
- Dimension of a mold or cavity.

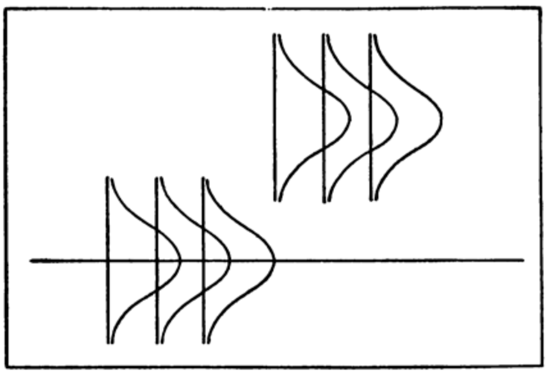


Fig. 147. Shift in level.

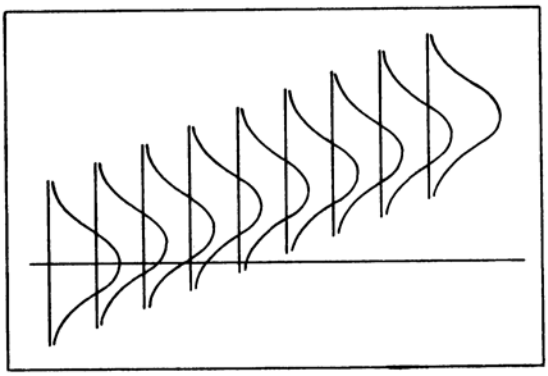


Fig. 148. Trend.

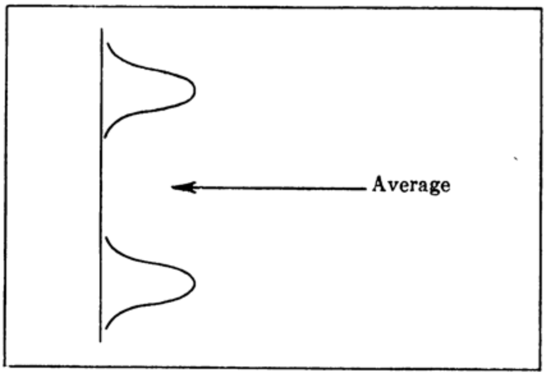


Fig. 149. Original average.

Winding tension on a reel.
Hardness of stock.
Supplier.
Calibration of a gage or test set.
Wear of tool.
Adjustment of the process or machine.
Expansion or contraction.
Aging.
Drift.
Humidity, moisture content, etc.
Misunderstanding of a drawing, or modification of a requirement.

Disturbances in the $\bar{X}$ chart (not associated with disturbances in the R chart) are almost invariably the result of causes similar to the above.

(2) *Indirect or "false" $\bar{X}$ causes.* There are three types of cause which can affect the $\bar{X}$ chart indirectly but are not true $\bar{X}$ causes. These causes show up on the R chart as well as the $\bar{X}$ chart and are in reality R-type causes. They appear on the $\bar{X}$ chart only as a reflection of the R chart. The engineer should carefully study the following:

A. The $\bar{X}$ chart can be affected by a *change in the proportion of distributions* which constitute a mixture. For example, Figure 149 shows a mixture of distributions and their original average. Figure 150 shows how the average is increased merely because there are fewer units in one of the component distributions. This type of cause can ordinarily be detected on the R chart, and should not be confused with the true $\bar{X}$ causes listed above.

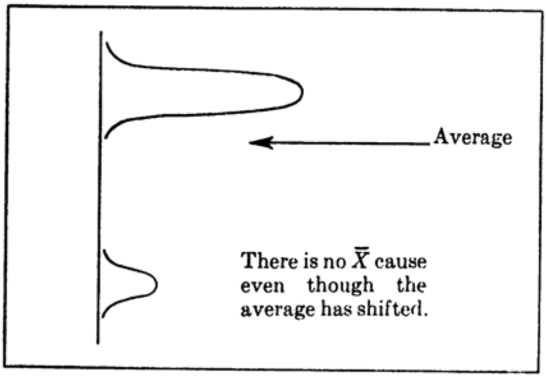


Fig. 150. Change in average.

B. Secondly, the presence of freaks in the data, or anything tending to create a pronounced skewness, will cause the $\bar{X}$ chart to follow the R chart and may throw the $\bar{X}$ chart out of control. See Figure 151.

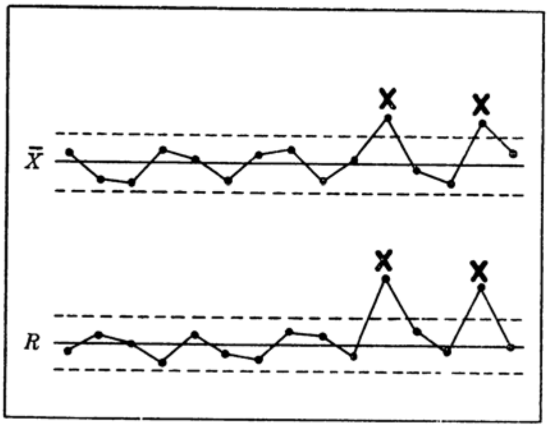


Fig. 151. $\bar{X}$ chart follows R chart.

C. Finally, if the level on the R chart increases or decreases with respect to its previously calculated limits, the control limits for the $\bar{X}$ chart will no longer be accurate, and therefore must not be used to determine whether the $\bar{X}$ points are out of control. See Figures 152 and 153 on the next page.

These three possibilities are the reason for the rules given repeatedly in this Handbook: "Do not attempt to interpret an $\bar{X}$ chart while the R chart is out of control." "Eliminate the R causes first and the chances are that the $\bar{X}$ causes will disappear along with them."

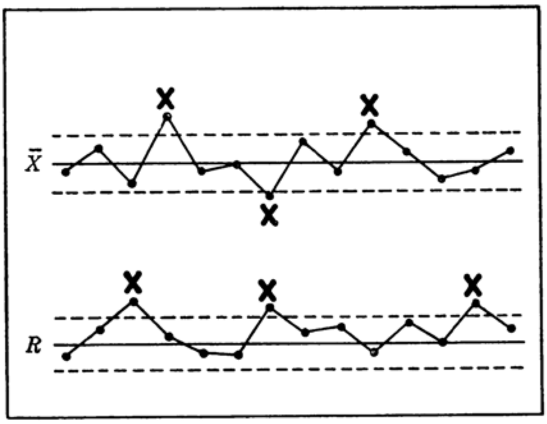


Fig. 152. $\bar{X}$ chart looks out of control but is not. Limits should be recalculated.

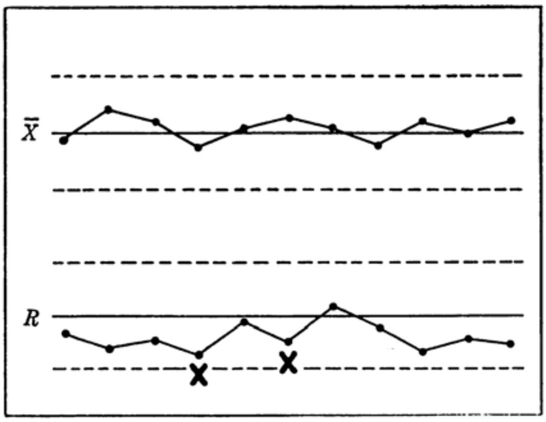


Fig. 153. $\bar{X}$ chart looks stratified but is not. Limits should be recalculated.

F-2.2 Mistakes to be avoided on $\bar{X}$ charts

The most common mistakes made in interpreting $\bar{X}$ charts are these:

(1) *Attempting to interpret an $\bar{X}$ chart when the R chart is not in control.* Examples of this were given above.

(2) *Attempting to relate the $\bar{X}$ chart to a specification without taking account of the distribution's shape.* In particular, be careful not to assume normality in a distribution which may have been truncated or screened.

(3) *Assuming that most of the product is at or near the "average."* Under ordinary circumstances this is a safe assumption, but the engineer should remember that it does not hold in all cases. It is possible for the product to consist of a bimodal or two-headed distribution with half of the product on the low side and half on the high side. In that case the average would be at the mid-point between the two portions of the product, and there might actually be no product on or around the average. See Figure 149 on page 153.

F-2.3 Summary

$\bar{X}$ charts are used to show trends, to indicate whether there is stability in the distribution's center, and under certain circumstances to indicate the relationship between the process and the specification. $\bar{X}$ charts should always be interpreted in conjunction with *R* charts, since mixtures or other abnormalities which show up plainly on the *R* chart are capable of causing apparent changes in level or pattern on the $\bar{X}$ chart.

F-3 INTERPRETATION OF R CHARTS

The *R* chart is a measure of uniformity or consistency. It reacts to a change in variation or spread. If one process is producing more uniform results than another process, the $\bar{R}$ for the first process will be lower. In general, we want the level on an *R* chart to be as low as possible.

If all units in the product are receiving the same treatment, the *R* chart will tend to stay in control. If the *R* chart does not remain in control or if its level rises, some units are receiving different treatment from the others. This may mean that a separate system of causes has been introduced, or there may be several different systems affecting different portions of the product at the same time. For example, instability of a test set or intermittent contacts on a relay or timer may result in the introduction of more than one system of causes.

If the level on the *R* chart rises and then stays in control at a higher level, it means that some new element has entered the cause system and has become a regular part of the process. Ordinarily an element which causes the *R* chart to rise will be an undesirable element. Examples of undesirable causes are: a change to a poorer quality material, increased production pressure, less competent operators or inspectors, less carefully designed tools and machines, a less adequate maintenance program, etc. The *R* chart will also rise if the component distributions in a mixture become more widely separated. This will tend to show up as stratification or mixture.

If the level of the *R* chart decreases and the chart then stays in control at the lower level, it means that some element which was treating the units differently has now been eliminated. For example, we have eliminated the poorly trained operators by re-training them; we have eliminated dirty pumps or sockets by a more careful maintenance program; we have eliminated the need for excessive play in the fixtures by getting better piece parts; we have reduced carelessness by installing control charts.

The *R* chart is far more sensitive to many important types of assignable cause than any other control chart. In particular the *R* chart is the best method of detecting mixtures, stratification, freaks, erratic conditions, interactions and general statistical instability. Since these conditions will seriously affect the engineer's interpretation of any other chart, the *R* chart must be considered the most important chart in a process capability study.

The principal conditions which the *R* chart is intended to reflect are shown in Figures 154, 155 and 156.

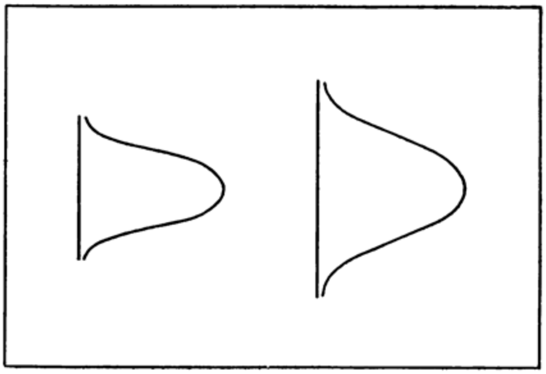


Fig. 154. Change in spread.

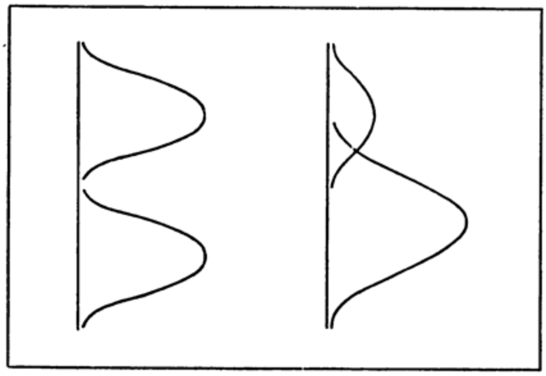


Fig. 155. Mixture of distributions.

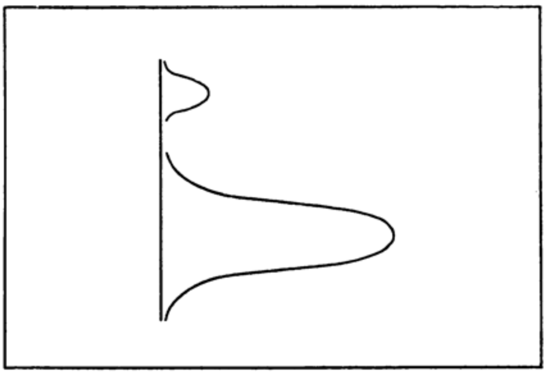


Fig. 156. Presence of freaks.

F-3.1 Causes affecting the R chart

All causes which affect an *R* chart have one element in common: that is, they are able to treat part of the product differently from the rest of the product. For example, a poorly trained operator does not do his work the same way every time, so part of the product receives different treatment. Similarly, a careless inspector does not insert his gage the same way every time; a machine in need of repair does not index the same way every time, etc. Causes which affect only a part of the product will change the spread of the distribution. The change in spread may or may not tend also to shift the center.

Among the causes which will affect an *R* chart are the following:

Poorly trained operator or inspector.
New operator or inspector.
Tired operator or inspector.
Material which is not uniform.
Fixture which is loose or has excessive play.
Machine in need of repair.

Something out of alignment.
Loose threads or screws.
Broken bolts.
Parts stuck together in barrel plating.
Testing equipment which is unstable.
Holders or fixtures which are warped.
Automatic controls which can go out of order.
Parts left at the end of a rod or strip.
Set-up parts.
Damage.
Careless handling.
Selective system of causes; for example, gold plating behaves differently when parts are chamfered than when they are burred.

In addition to the above, the R chart will be affected by mixtures of different "lots." The engineer should remember this in obtaining data for a process capability study, particularly in view of the fact that his control limits and other estimates will be based on the R chart.

Occasional freaks, "wild units," or "mavericks" will show up as isolated high points on the R chart. These are easily recognized because they are so different from the rest of the pattern.

F-3.2 Summary

R charts are used to show the magnitude of the spread of the process being studied, to indicate whether the spread is stable, and to reveal information associated with mixtures, interactions and various forms of instability. R charts should always be interpreted before the corresponding $\bar{X}$ charts are interpreted, in view of the importance of this type of information.

F-4 JOINT INTERPRETATION OF $\bar{X}$ CHARTS AND R CHARTS

Since $\bar{X}$ charts and R charts are concerned with different phases of the distribution being studied, the two charts should finally be interpreted in conjunction with each other. There are two reasons why this is important.

(1) The charts re-inforce each other in giving information about the distribution. It is necessary to consider both center and spread if the information is to be useful.

(2) By considering the charts together it is possible to obtain new information which was not obtainable from either chart considered separately. The additional information has to do with peculiarities or abnormalities in the shape of the distribution from which the samples are being taken.

The basis for obtaining this additional information is as follows:

When samples are taken at random from a normal distribution, there is no correlation between the $\bar{X}$ and R values. That is, the fact that an $\bar{X}$ value is high does not tend to make the R value high or vice versa. There is no necessary relationship between averages and ranges.

Consequently, when a series of samples are plotted on a control chart, if those samples came from a normal population, the $\bar{X}$ points and R points do not tend to follow each other. Both of the patterns appear to be unrelated or truly "random."

If samples are taken from a very skewed population, there will be definite correlation between the $\bar{X}$ points and the R points. If the population is skewed in such a way that the long tail is on the high side (positive skewness) there will be positive correlation between $\bar{X}$ and R. The $\bar{X}$ points will tend to follow the R points, moving in the same direction.

If samples are taken from a population which is skewed with its long tail toward the low side (negative skewness) there will be negative correlation between the $\bar{X}$ points and the R points. The $\bar{X}$ chart will tend to become an "inverted image" of the R chart. The $\bar{X}$ points tend to follow the R points, but in the opposite direction.

The greater the skewness, the more definitely the points will tend to follow each other.

Do not confuse *changes in level* on the $\bar{X}$ chart and R chart with the "tendency to follow" which is being discussed here. The "tendency to follow" means that the *individual points* move up and down together, not the general level. See pages 176–177.

In brief, the engineer should be alert for any indications on the separate charts or on the $\bar{X}$ and R combination which tend to show that the patterns are behaving in anything but a random manner.

F-5 INTERPRETATION OF p-CHARTS AND OTHER ATTRIBUTES CHARTS

A p-chart shows the proportions into which a distribution has been divided. Frequently a p-chart is used to represent "percent defective," and the distribution has been divided into two parts, defective and non-defective, by a simple process of comparing the units of product with a specification and then classifying them in one or the other of these two groups. See Figure 157.

However, p-charts can be used to represent any proportion and need not be associated with product which is defective. For example, a p-chart can be used to show the proportion of units which fall within a certain voltage range as compared with the proportion which fall within other voltage ranges. In all cases, the p-chart represents a division of the distribution on the basis of some previously determined system of classification.

p-Charts can be combined or sub-divided at will; that is, the proportion represented may be the proportion classified on the basis of a single characteristic only, or it may be the proportion with respect to a number of characteristics taken together. The interpretation of a p-chart depends to a considerable extent on knowledge of the number of characteristics which formed the basis for the classification. This is particularly important in the case of a process capability study where early p-charts appear to show fairly good control. If the p-chart results from a combination of many characteristics, the apparent control may be reflecting a "statistical balance" among these characteristics rather than the "singleness of a cause system" which is the real measure of process capability.

The conditions which a p-chart is intended to reflect are shown in Figures 158 and 159.

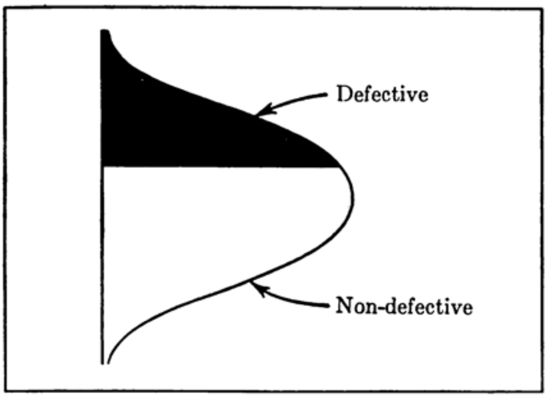


Fig. 157. Basis for a p-chart: some form of classification.

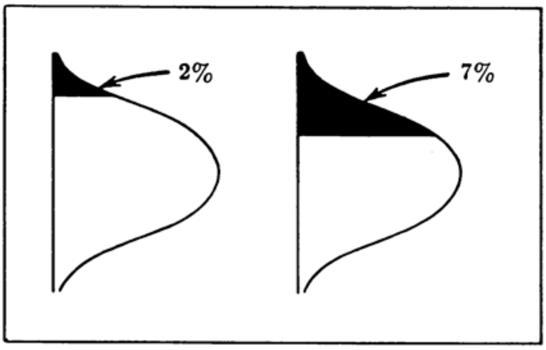


Fig. 158. Change in percent defective (or other basis for classification).

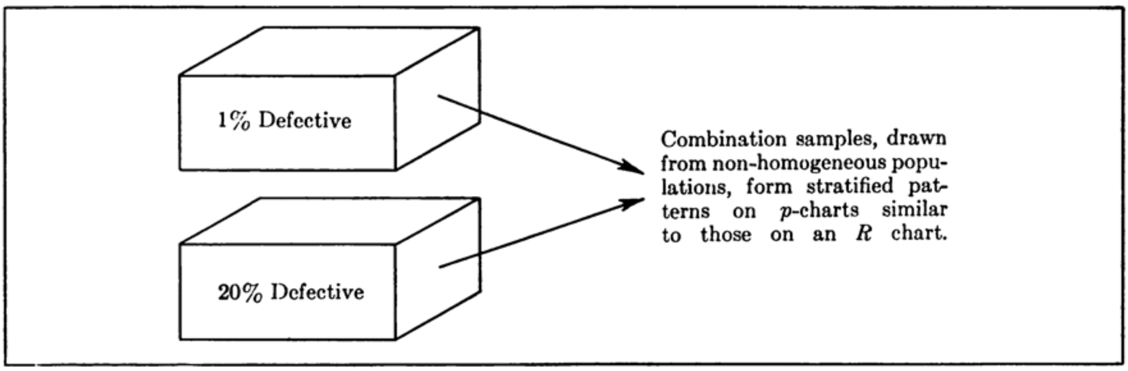


Fig. 159. Stratified sampling.

F-5.1 Causes affecting the p-chart

The p-chart does not reflect any characteristic of the distribution directly. That is, it does not indicate the average, shape or spread. It shows simply the arbitrary classification of the distribution into two or more parts. For this reason it is harder to identify specific causes or types of cause as likely to affect the p-chart.

The causes for out-of-control conditions on p-charts may include anything which is capable of affecting the center, spread or shape of the product distribution. The p-chart is also very sensitive to causes which affect the standards being used as the basis for classification.

In tracing the causes affecting p-charts the engineer should rely heavily on job knowledge. He should check the surrounding conditions which are associated with the chart and investigate the process elements which he believes might contribute to those conditions. Among the elements which may be investigated are the following:

(1) Materials (including both processing and inspection).

(2) Machines (including tools, fixtures, gages and other facilities).

(3) Methods (including layouts and other information, deviations from prescribed procedures, changes in motion patterns or changes in the operators' "efficiency").

(4) Men (including their training, attitudes and experience, whether they are properly instructed, whether they are using control charts).

Throughout any investigation into the causes affecting p-charts, the engineer should remember that large variables may be operating in the process from time to time without showing up on this chart. For example the distributions in Figure 160 may all look alike on a p-chart because they all have the same percent defective.

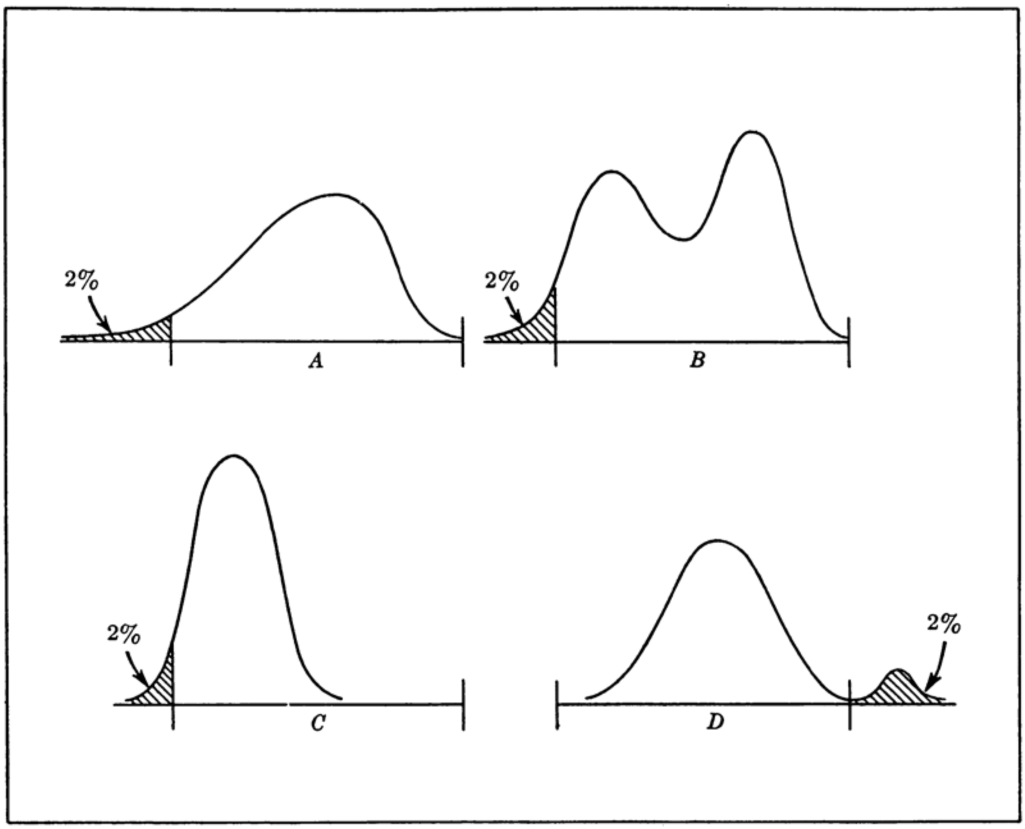


Fig. 160. Distributions 2% defective.

F-5.2 Mistakes to be avoided on p-charts

The most common errors made in interpreting p-charts are the following:

(1) *Assuming too quickly that a p-chart is in control.* As mentioned above, a p-chart which apparently shows control may only be reflecting a state of balance. Before concluding that an overall p-chart is actually in control, break it down into the component characteristics or sources of product and study the sources separately. If the sources are not separately controlled, it is necessary to modify any conclusions based on the overall chart.

(2) *Failing to take account of a change in standards.* One of the first things to look for in a p-chart is a possible change in the criteria or in the process checker's method of applying the criteria. If the checker fails to look for certain defects, the level on the p-chart may drop. This is much more likely to happen on a p-chart than on an $\bar{X}$ and R chart, because of the fact that many characteristics may be combined on a single p-chart.

The engineer should also be alert to the fact that slight changes in the line of demarcation between "defective" and "nondefective" may make surprisingly large differences in percentage. Changes in the calibration or maintenance of test equipment may cause large fluctuations on the chart if the dividing line which is used as a basis for classification happens to come at a steep part of the distribution. See Figures 161 and 162.

The p-chart contains no mechanism for revealing that the causes in these two cases were really of equal magnitude.

(3) *Concentrating on defects which show a high $\bar{p}$ to the exclusion of defects which show a lack of control.* In analyzing p-charts in process capability studies it is common practice to have a number of p-charts on various characteristics plotted on similar scales so they can easily be compared. The engineer frequently tends to pay attention to the p-charts which show a high level of $\bar{p}$ and to ignore p-charts which show a lower level, even though the latter may be seriously out of control.

Since the object of a capability study is to obtain as much information as possible about the process and its variables, the engineer should keep in mind that out-of-control patterns are an excellent source of engineering information. Tracing the causes of unstable patterns on the defects which tend to occur less frequently has often led to reducing the level of other p-charts.

F-5.3 Summary

p-Charts are used to show the general level of a process in terms of percent defective or some other proportion, to indicate overall trends, and to permit easy comparison between operators or machines by means of individual charts. When p-charts are to be compared it is necessary to make sure that the base for comparison is consistent. The principal difficulties in interpreting p-charts arise from including too many characteristics on one chart.

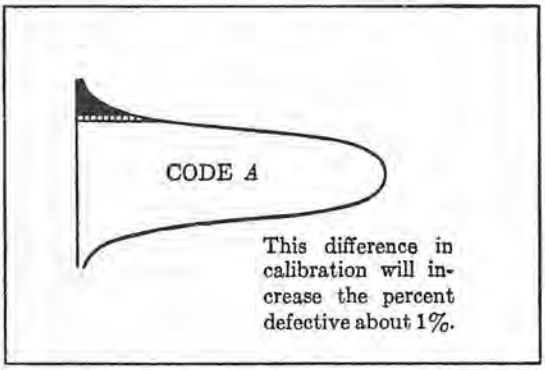


Fig. 161. Effect of recalibrating a test set: Code A.

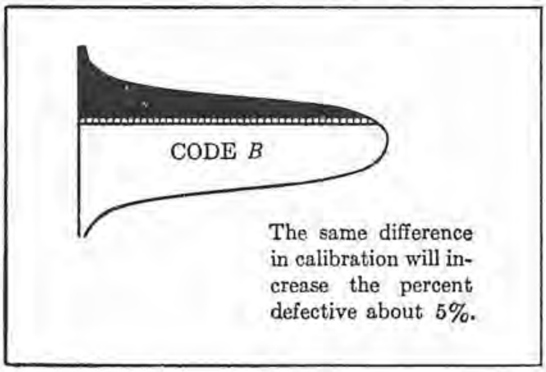


Fig. 162. Effect of the same test set recalibration: Code B.

F-5.4 Special note on attributes control charts

The information given on pages 157–159 applies to all attributes control charts, including *np*-charts, *c*-charts and *u*-charts. All attributes charts are interpreted in the same way as *p*-charts, and the same precautions should be observed in deciding how many characteristics to include and in drawing conclusions.

F-6 INTERPRETATION OF A CHART FOR INDIVIDUAL MEASUREMENTS

This chart shows the fluctuations of individual measurements in some predetermined order of plotting. The basis for determining the order is frequently time, but in special analyses, including designed experiments, the order may correspond to sources of data, codes, type of product, or some other basis for identification. The chart of individuals is used to show:

a. General trends.
b. Fluctuations of unusual magnitude.
c. Clustering of abnormal measurements at certain points.
d. The relationship between the individual measurements and some previously established standard or specification.

Many of the conditions which show up on an $\bar{X}$ and R chart can also be detected, somewhat less precisely, on the chart of individuals. The tests for unnatural patterns are less reliable than on an $\bar{X}$ and R chart, since the individuals chart may be seriously affected by any change in the shape of the distribution. It is sometimes advisable to check the conclusions drawn from a chart of individuals by obtaining more data and plotting an $\bar{X}$ and R chart.

On the other hand there are certain types of information which may show up more plainly on an individuals chart than on an $\bar{X}$ and R chart. Among these are the following:

(1) **Cycles** (regular repetition of pattern)

Short cycles, in particular, may not show up for some time on an $\bar{X}$ and R chart.

(2) **Trends** (continuous movement up or down)

These may show up more rapidly on a chart of individuals. On the other hand it is easy to see many apparent trends which do not actually exist.

(3) **Mixtures** (presence of more than one distribution)

This shows up on the chart of individuals in much the same way that it shows up on an R chart. That is, there is an absence of points near the middle of the pattern with excessive numbers of points toward either edge. On the chart of individuals this can often be detected by the fact that lines connecting the individual dots tend to be long and rather similar in length instead of showing a random mixture of long and short lines intermingled with each other.

(4) **Grouping or bunching** (measurements clustering in spots)

If all the freaks on the high side tend to occur at one or two places instead of being scattered randomly throughout the data, this may be detected more promptly on the individuals chart than on an $\bar{X}$ and R chart. The individuals chart may also show other peculiarities in the data—for example, that the measurements tend to occur in pairs.

(5) **Relation between the general pattern of individuals and the specification**

This chart shows plainly whether the individuals plotted are in or out of limits; whether they are well centered between specifications or close to one side.

F-6.1 Causes affecting the chart for individual measurements

The individuals chart can be affected by any of the causes which affect either $\bar{X}$ and R charts or *p*-charts. While it is not possible to distinguish between $\bar{X}$ disturbances and R disturbances with anything like the precision which is possible on $\bar{X}$ and R charts, nevertheless the eye can pick up many visual impressions of changes which affect either the center or spread.

F-6.2 Mistakes to be avoided on a chart for individual measurements

On a chart for individuals the sample size is 1, and the control limits are the same as would be used on an $\bar{X}$ chart where $n = 1$. The "$\sqrt{n}$" relationship still holds in comparing the control limits with a specified maximum or minimum limit (see pages 30–31), but in this case (since $n = 1$) the control limits may actually coincide with the specification.

In considering the centerline on a chart for individuals, remember that the shape of the distribution is very important in determining what portion of the product will exceed the specified limits. If the distribution is skewed in the direction away from a certain limit it is safe to run closer to that limit, and vice versa.

F-6.3 Summary

Charts of individual measurements are intended to convey the same general type of information as an $\bar{X}$ and R chart. The control limits are, in general, less sensitive and precise. The chart must be interpreted with considerably more caution. Where necessary check the conclusions on the individuals chart by making an $\bar{X}$ and R chart.

F-7 ANALYSIS OF PATTERNS

The following Sub-section contains descriptions of 15 common control chart patterns, arranged in alphabetical order by name.

1. Cycles.
2. Freaks.
3. Gradual change in level.
4. Grouping or bunching.
5. Instability.
6. Interaction.
7. Mixtures.
8. Natural pattern.
9. Stable forms of mixture.
10. Stratification.
11. Sudden shift in level.
12. Systematic variables.
13. Tendency of one chart to follow another.
14. Trends.
15. Unstable forms of mixture.

Each pattern is explained in a short verbal description and is illustrated by a typical drawing of a control chart. Wherever possible, the underlying distributions represented by the pattern are also shown. Beneath each pattern are listed four types of control chart—$\bar{X}$, R, p and individuals—and a typical list of causes likely to be associated with each chart. This information can be used in interpreting a control chart as follows:

(1) From inspection of the control chart, decide which type of pattern the chart represents.

(2) Look up this pattern in the following pages and compare the chart with the illustrative drawing.

(3) Study the verbal description of the pattern, and relate its description to what is known about your process. Select the appropriate list of causes—$\bar{X}$, R, p or individuals—and attempt to think of similar causes which may be operating in your process.

For an example of the manner in which this information is used, see pages 66–71.

In addition to the information available from patterns, there are many other practical aids in interpreting control charts. Some of these are given in the Engineering Section, pages 53–56 and 61–65. Others can be found in the Shop Section, pages 189–190 and 217–219.

F-8 CYCLES

Cycles are short trends in the data which occur in repeated patterns. Any tendency of the pattern to repeat, by showing a series of high portions or peaks interspersed with low portions or troughs, is an indication of an assignable cause, since the primary characteristic of a random pattern is the fact that it does not repeat. The causes of cycles are processing variables which come and go on a more or less regular basis. In the case of machines they may be associated with a succession of movements, positions or heads. In the case of manually controlled operations, they may be associated with fatigue patterns, shipping schedules, conditions affecting the day and night shifts. In some types of product they may be associated with seasonal effects which come and go more slowly.

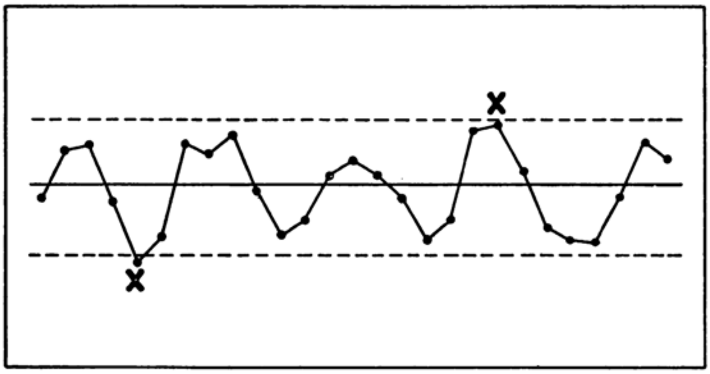


Fig. 163. Cycles.

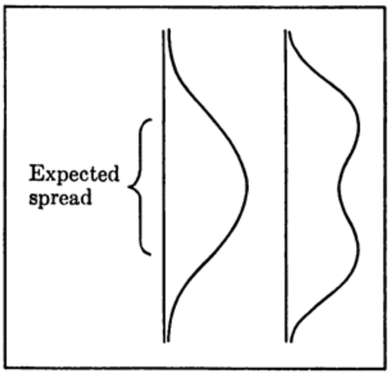


Fig. 164. Distributions associated with cycles.

The underlying distribution is wider than would be expected from an R chart. It may be bimodal, showing two humps or peaks. Cycles are identified by determining the time interval at which the successive peaks or troughs appear and relating this interval in some manner to the elements in the process. For example:

- An engineer discovered that every seventh measurement was lower than the others. He was able to relate this to a fixture containing 7 holes.
- A supervisor found that every third sample on a continuous piece of wire was suspiciously high. He knew that the samples had been taken 25″ apart. He was able to relate this to a disc over which the wire passed which was 75″ in circumference.
- Cyclical patterns in riveting operations were found to be related to times of the day. The operator's technique varied according to the beginning or end of the week, lunch periods, rest periods, change in shifts, etc.

Other causes which can create this type of pattern are as follows:

$\bar{X}$ Chart

(*R* chart must be in control.)

Seasonal effects such as temperature and humidity.
Worn positions or threads on locking devices.
Roller eccentricity.
Operator fatigue.
Rotation of people on job.
Difference between gages used by inspectors.
Voltage fluctuation.
Regular difference between day and night shifts.

R Chart

Maintenance schedules.
Operator fatigue.
Rotation of fixtures or gages.
Regular difference between day and night shifts.
Wear of tool or die (causing excessive play).
Tool in need of sharpening (causing burrs, etc.).

p-Chart

Sorting practices.
Sampling practices.
Regular difference between suppliers.

Individuals Chart

Any of the causes affecting $\bar{X}$ charts or R charts.

F-9 FREAKS

Freaks result from the presence of a single unit or a single measurement greatly different from the others. Such units are generally produced by an extraneous system of causes. Occasionally, however, the measurements that look like freaks are in reality a normal part of the process. For example, dielectric breakdowns may actually be the long tails of a distribution of dielectric strength. In this case the "freaks" are a matter of degree.

Another common source of freaks is a mistake in calculation. Failure to subtract properly in obtaining the R point, or failure to divide by the proper number in calculating an $\bar{X}$ or p value, will sometimes have this effect.

Occasionally an apparent freak is the result of a plotting error, as when the person plotting the point has misinterpreted the scale. Accidental damage or mis-handling may also result in freaks.

Freaks are among the easiest of the patterns to recognize, and it is also simple in most cases to identify the cause. The mere fact that the freak is so different from other units in the product makes the identification simpler.

Typical causes which can create this type of pattern are the following:

$\bar{X}$ Charts

(*R* chart must be in control.)

Freaks do not ordinarily show up on an $\bar{X}$ chart without a corresponding indication on the *R* chart. A possible exception is the case where a sudden abnormal condition in the process may affect all or most of the units in the sample. Among such conditions may be the following:

Wrong setting, corrected immediately.

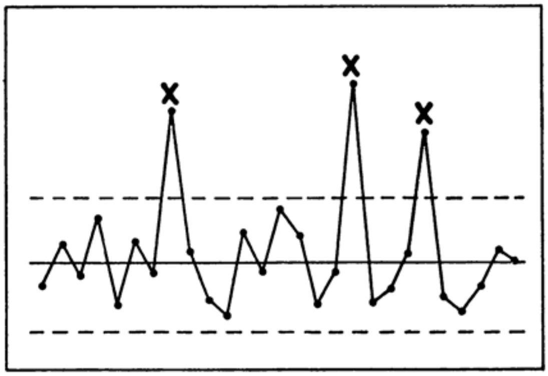


Fig. 165. Freaks.

Error in measurement.
Error in plotting.
Data obtained on a non-linear scale (logarithmic or exponential.) For example, insulation resistance.
Incomplete operation.
Omitted operation.
Breakdown of facilities.
Accidental inclusion of experimental units.

R Chart

Accidental damage in handling.
Incomplete operation.
Omitted operation.
Breakdown of facilities.
Experimental unit.
Set-up parts.
Error in subtraction.
Occasional parts from end of a rod or strip.
Measurement error.
Plotting error.
Some obvious physical abnormality which can be detected by examining the units in the sample that produced the freak point.

p-Chart

Variations in sample size.
Sampling from a distinctly different distribution.
Occasional very good or very bad lot.

Individuals Chart

Same as *R* chart.

Occasionally freaks result from the fact that the characteristic being plotted has a non-

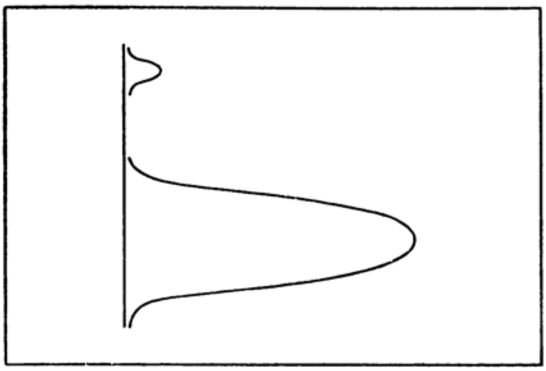


Fig. 166. Distribution associated with freaks (bimodal).

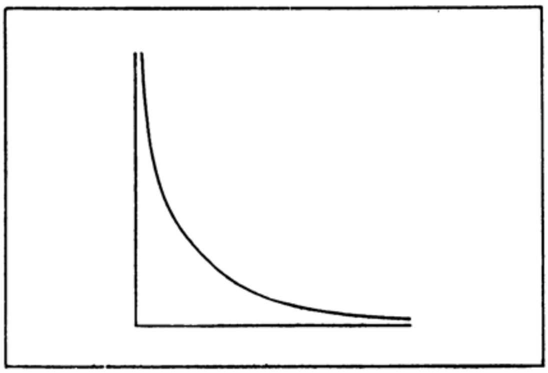


Fig. 167. Distribution associated with freaks (L-shaped).

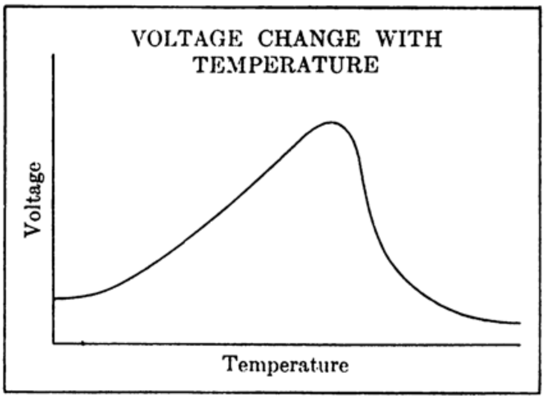


Fig. 168. Source of apparent freaks.

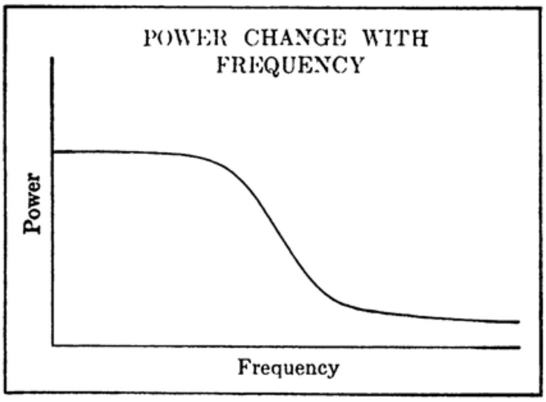


Fig. 169. Another source of apparent freaks.

linear behavior. For example, a characteristic may rise sharply at a certain temperature or pressure: or it may drop off abruptly in a "shoulder" as shown in Figures 168 and 169. If the process is running near the steep slope of such a curve, so that several units in the sample reflect the high part and some other unit or units reflect the drop, this may show up as a freak.

F-10 GRADUAL CHANGE IN LEVEL

A gradual change in level will ordinarily indicate one of two things:

(1) There is some element in the process which is capable of affecting a few units at first and then more and more as time goes on. For example, a group of new operators has been added. As the operators become better trained (which happens at varying rates of speed) more and more of the distribution is affected. The same thing can happen when newly designed fixtures are being introduced one by one, when poorly controlled lots from the storeroom are being replaced by better controlled lots, when a maintenance program is gradually being extended to cover more and more equipment, when operators begin to follow their control charts more and more closely, etc. After the change has taken place the chart tends to settle at some new level.

(2) It may be that some element in the process has been changed abruptly, but because of the amount of product "going through the mill" it shows up gradually at the later operations. This could be any one of the causes mentioned under "Sudden Shift in Level."

In either case a gradual change in level produces patterns like the one in Figure 170. The total distribution, including both levels, is wider than would be expected from an R chart.

Gradual changes which do *not* tend to settle down to a new level are spoken of as "trends." Gradual changes in level (with the change occurring in the direction of improvement) are very common in the early stages of a quality control program.

Typical causes which will create this type of pattern are the following:

$\bar{X}$ Chart

(**R chart must be in control.**)

Gradual introduction of new material, better supervision, greater skill or care on the part of the operator.
Change in maintenance program.
Introduction of process controls in this or other areas.

R Chart

Change to lower level:
Better fixtures.
Better methods.
Greater skill or care on the part of the operator.
Change to higher level:
Conditions opposite to the above.

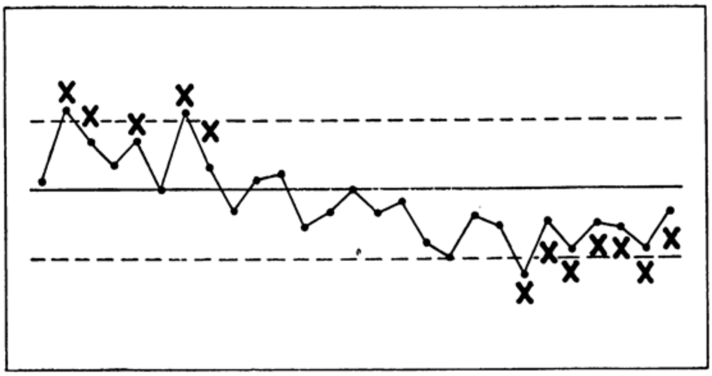


Fig. 170. Gradual change in level.

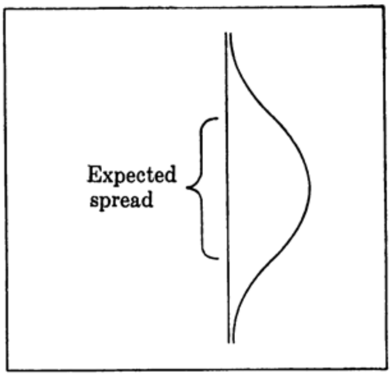


Fig. 171. Distribution associated with gradual change in level.

p-Chart

Any of the causes affecting $\bar{X}$ and R charts.
Addition or removal of requirements.
Relaxation or tightening of standards.

Individuals Chart

Same as $\bar{X}$ chart.

F-11 "GROUPING" OR "BUNCHING"

One of the characteristics of a natural pattern is that measurements of any given magnitude tend to be scattered more or less uniformly throughout the data. It is an indication of unnaturalness if all or most of the similar measurements occur quite close together. When measurements cluster together in such a non-random fashion it indicates the sudden introduction of a different system of causes. For example, the low points in Figure 172 came from a pan of rejected parts which were shipped accidentally. A similar pattern was obtained on a disc-spraying job when the mask occasionally slipped and permitted conductive material to leak over the edge of the discs. The underlying distribution is a mixture, frequently showing a few units distinctly separated from the rest of the product.

Indications of this kind are sometimes observed on $\bar{X}$ charts but they tend to occur more frequently on charts for individuals, R charts or p-charts. In many cases a chart of individual measurements will be more sensitive in picking up this type of disturbance than any other chart.

Typical causes for non-random bunching of the measurements are as follows:

$\bar{X}$ Charts

(R chart must be in control.)

Measurement difficulties.
Change in the calibration of a test set or measuring instrument.

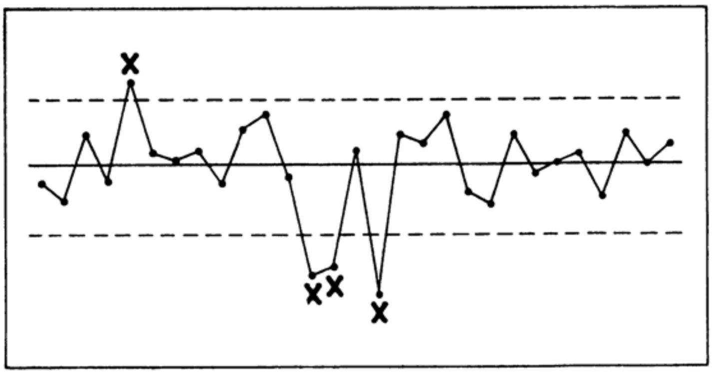


Fig. 172. Grouping or bunching.

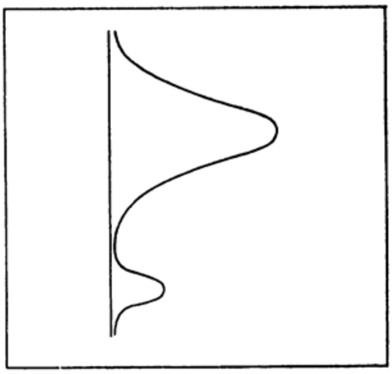


Fig. 173. Distribution associated with grouping or bunching.

Different person making the measurements.
Shift in distribution for a limited period.

R Chart

Freaks in the data.
Mixture of distributions.

p-Chart

Change in technique of classification.
Shift in one of the underlying distributions of product.
Changes in assortment of product.

Individuals Chart

Extraneous cause resulting in a totally different distribution for a limited period of time.
Errors in plotting.

F-12 INSTABILITY

Instability of the pattern is shown by unnaturally large fluctuations. The pattern is characterized by erratic ups and downs, frequently resulting in x's on both sides of the chart. The fluctuations of the pattern appear to be too wide for the control limits. This type of pattern may arise in either of two ways:

A. A single cause, capable of affecting the center or spread of the distribution, may operate on the process erratically.
B. A group of causes, each capable of shifting the center or spread (or both), may operate on the process in conjunction with one another.

In the latter case the patterns of instability may become very complex. The causes may be more difficult to identify than the causes of simpler patterns. The underlying distribution is wide and frequently irregular in shape. It may exhibit several peaks.

Instability in a process is frequently associated with mixtures, and "Unstable Mixtures" may be regarded as a special form of "Instability." There are two ways of discovering the causes of complex instability:

(1) Check the process for obvious "Unstable Mixtures" as explained on pages 179–180. These are the easiest causes of instability to identify and eliminate. When the unstable mixtures are eliminated, the pattern of instability may become much easier to interpret.
(2) If the pattern is still complex, break the process into smaller segments or operations and plot a separate chart for each. Take the one whose pattern is most similar to the original complex pattern, and break it down still further. Continue in this way until the pattern becomes simple enough to interpret.

In seeking out the causes of complex patterns, remember that the ultimate causes are likely to be very simple. They appear to be complicated only because they exist in complex combinations. Among the common causes for instability are the following:

$\bar{X}$ Chart

(*R* chart must be in control.)

Simple causes

Overadjustment of a machine (where the

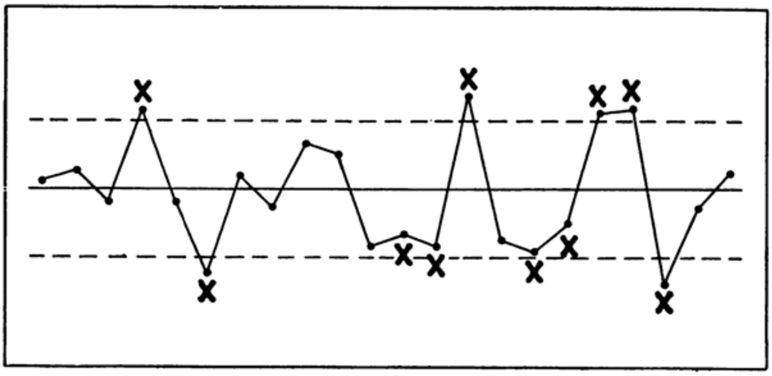


Fig. 174. Instability on an $\bar{X}$ chart. On an *R* chart the pattern is similar, but the low points tend to gather just inside the lower limit, since they cannot fall below it.

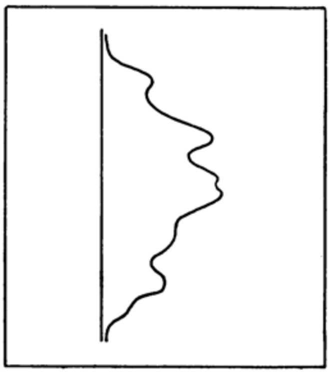


Fig. 175. Distribution associated with instability.

operator resets on the basis of one or two measurements instead of using a chart).

Fixtures or holders not holding the work in position.

Carelessness of operator in setting temperature control or timing device.

Different lots of material mixed in storeroom.

Piece parts mixed on the line (in different bins or pans).

Code differences (which are related to differences in design or in difficulty of assembly).

Differences in test sets or gages.

Shop running deliberately on high or low side of specification (causing other distributions to be run off-center also).

Erratic behavior of automatic controls.

Complex causes

Effect of many process variables on an end characteristic.

Effect of screening and sorting operations at various stages in the process.

Effect of differences in testing or gaging methods after product has been accumulated for shipment.

Effect of experimental or development work being done by engineers.

In general, we attempt to keep complex causes from affecting the $\bar{X}$ chart by locating $\bar{X}$ and R charts as far back in the process as possible.

Note also that apparent instability on the $\bar{X}$ chart frequently accompanies an R chart that is out of control. In such cases the $\bar{X}$ chart may appear to have erratic ups and downs even when the center of the distribution is actually stable. See pages 153–154.

R Chart

Instability on high side

Untrained operator.

Too much play in positioning or holding fixture.

Mixture of material.

Machine in need of repair.

Unstable testing equipment.

Work holder warped.

Lapping plates worn.

Lapping materials not properly used.

Operator carelessness.

Assemblies off-center.

Defective piece parts.

Trouble with test set.

Instability on low side

Better operator.

More uniform piece parts.

Better work habits.

Possibly the effect of control charts installed in other areas.

p-Chart

High side

Operator inexperience.

Operator carelessness.

Poor maintenance.

Defective piece parts or material.

Trouble with test set.

Low Side

Operator improvement.

Better sub-assemblies.

Better equipment or material.

Relaxation of standards.

Improper checking.

Other causes

Instability on the p-chart may also be caused by:

- Variations in sample size.
- Occasional lots which are very good or very bad.
- Sampling from distinctly different distributions.
- Non-random sampling.

Individuals Chart

Any of the causes which can affect the $\bar{X}$ chart or R chart.

F-13 INTERACTION

Interaction is the tendency of one variable to alter the behavior of another; the tendency of two or more variables to produce an effect in combination which neither variable would produce if acting alone. Interactions are studied formally by means of designed experiments. They are also detected informally by means of process capability studies.

Interactions may be detected on an $\bar{X}$ chart whenever the data have been identified in two or more ways. See Figure 176 on page 168.

In addition, interactions due to variables not

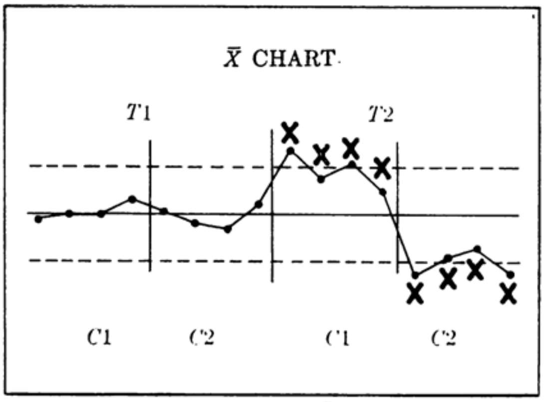


Fig. 176. Interaction on the $\bar{X}$ chart: there is an interaction between T and C.

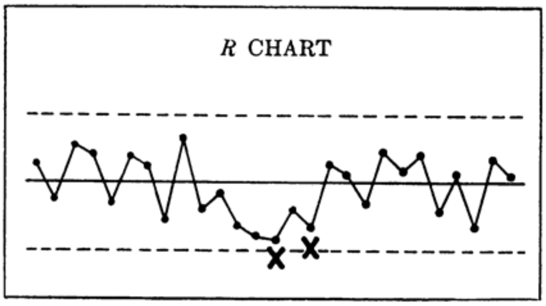


Fig. 177. Interaction on the *R* chart.

previously identified can often be detected on the *R* chart. See Figure 177.

The chart in Figure 177 is explained as follows:

All variability in a process can be thought of as the result of interactions. That is, potentially large variables exist in the process and tend to occur at more than one level. The effects of these are modified by other variables which also exist at different levels. Machine effects are modified by type of maintenance or material. Operator effects are modified by amount and kind of training. Effects due to manual skill are modified by differences in tools, tweezers or gages. It is difficult to think of any process variable which is not, in reality, the result of interactions.

In a designed experiment, where certain variables can be deliberately removed for analysis, the effects of all other variables are treated as "unanalyzed interaction." They are included in the residual or "experimental error." In the same way, in a process capability study, the unanalyzed interactions are included in the *R* chart. If significant variables exist and are present at more than one level, they tend to inflate the *R* chart. If, by intention or otherwise, one of the significant variables should occur at one level only, this would immediately remove some of the inflation from the *R* chart.

A low pattern on the *R* chart, like that in one portion of Figure 177, indicates that some of the inflation usually present in the process has temporarily been removed. We conclude from this that some important interacting variable must have been present at one level only.

The chart in Figure 177 shows that this variable, if kept permanently at one level, could reduce the spread of the process to less than half its usual magnitude.

Low x's on the *R* chart are one of the most important indications which can occur in a process capability study.

Identifying the interacting variables

Since low x's on the *R* chart indicate that some important variable was present at only one level, this can be used to identify the important interactions.

- If the samples that produced the low x's were all from one inspector, the inspectors are likely to be an important interacting variable.
- If the samples that produced the low x's were all from one fixture, the fixtures are likely to be an important interacting variable.
- If the samples that produced the low x's were all from newly lapped fixtures, the uneven or warped surfaces of the older fixtures may be the important interacting variable. And so on.

Remember that the spread can usually be reduced if any *one* of the interacting variables is reduced to a single level. If one of the variables cannot be reduced to a single level economically, try reducing another. Some practical examples of this are the following:

Machines of different ages may constitute an important variable, but only in the presence of different degrees of maintenance. (There is an interaction between machine age and maintenance). Since it would not be economically feasible to run the job with only one machine, we concentrate on eliminating the differences in the effectiveness of the maintenance.

Similar interactions may exist between operator differences and different degrees of training or supervision.

From this point of view, one of the chief objectives of a quality control program is to improve the piece parts, improve the design of the tool, provide better training or closer supervision, so that we can use different operators, machines and batches of material and still get uniform product.

For formal methods of studying interaction in a designed experiment, see pages 94–97 and 99–101.

F-14 MIXTURES

In a mixture pattern the points tend to fall near the high and low edges of the pattern with an absence of normal fluctuations near the middle. This pattern can be recognized by the unnatural length of the lines joining the points, which tends to create a more or less obvious "seesaw" effect. See Figure 178.

A mixture pattern is actually a combination of two different patterns on the same chart—one at a high level and one at a low level. If we were to take an extreme mixture pattern and reconnect the points in a different manner they would look like Figure 179. If we grouped the plotted points into a frequency distribution they would tend to look like Figure 180.

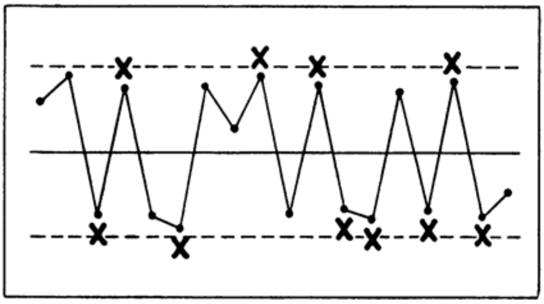


Fig. 178. Mixture.

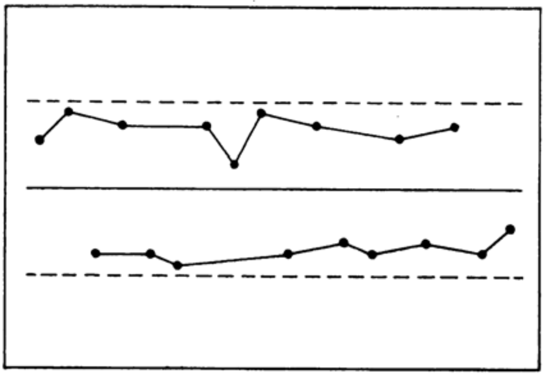


Fig. 179. Separate patterns.

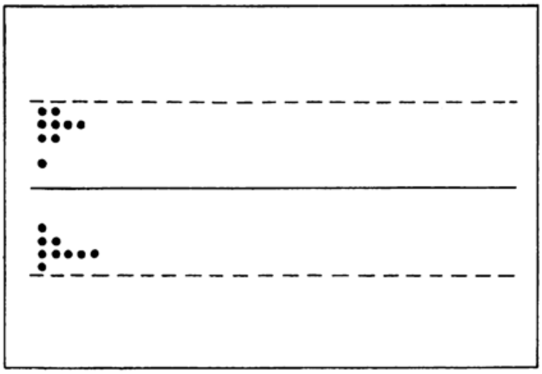


Fig. 180. Separate clusters of points.

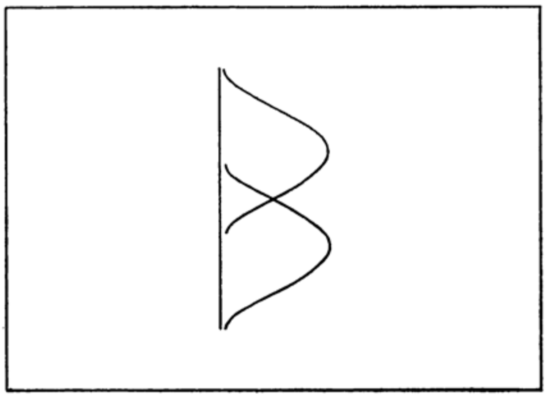


Fig. 181. Distribution associated with mixture (obvious mixture).

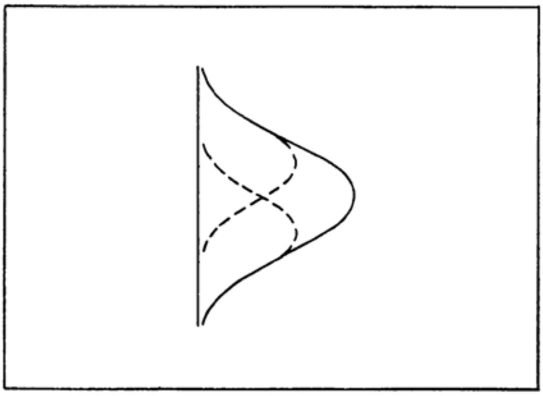


Fig. 182. Distribution associated with mixture (less obvious mixture).

The components in a mixture may be widely separated as in Figure 181 or close enough to blend as in Figure 182. The wider the separation between the component distributions, the more obvious will be the indications of mixture.

When the component distributions maintain the same relative positions and proportions over a period of time, we call it a "Stable Mixture." When the relative positions or proportions are not constant, we call it "Unstable Mixture." Since the causes of stable and unstable mixtures are not the same, these two types of mixture are listed below as separate patterns. See pages 171–172 and 179–180.

F-15 NATURAL PATTERN

A natural pattern is one which gives no evidence of unnaturalness over a long series of plotted points. The pattern is stable; there is no trend; there are no sudden shifts, no erratic ups and downs, no x's. The cause system appears to be in balance and the process is "in control."

Stability alone, however, is not sufficient reason for calling the pattern natural. A stratification pattern may have stability, but it shows definite evidence of assignable causes.

The physical characteristics of a natural pattern are described on page 24.

The distribution associated with a natural pattern is likely to be fairly smooth and unimodal, not extremely flat, not extremely skew. However, a natural pattern does not necessarily indicate a "normal" distribution.

The following is a summary of the principal meanings of a natural pattern:

R chart

A natural pattern on the R chart provides direct evidence of the process uniformity. If the chart covers an operator's work it shows the operator's technique; this can be used to compare different operators. If the chart covers a machine dimension it shows the capability of the machine: that is, how close a tolerance it can hold. This can be used to compare different machines. It is also a direct measure of the spread of the underlying parent distribution. See page 56.

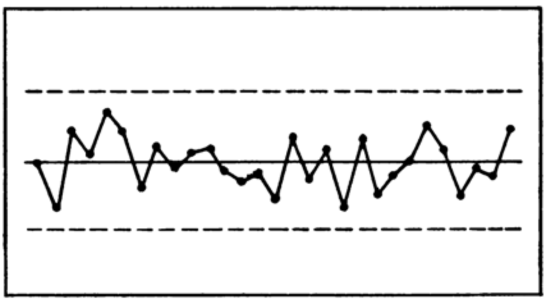


Fig. 183. Natural pattern.

$\bar{X}$ chart and R chart

A natural pattern on both the $\bar{X}$ chart and the R chart gives direct evidence of the average of the parent distribution. It also means that the average did not change during the charted period and that most of the product was actually near the indicated average. When the $\bar{X}$ and R chart are both in control it is possible to make reliable comparisons between the process and the specified limits. See pages 119–122.

p-Chart

A natural pattern on a p-chart indicates that there is a constant fraction defective in the product; also that the sampling is random (not stratified).

Chart for individual measurements

A natural pattern on a chart for individual measurements indicates that the distribution is stable with respect to both average and spread; also that its shape is reasonably symmetrical, since unsymmetrical distributions tend to give indications of unnaturalness on this type of chart.

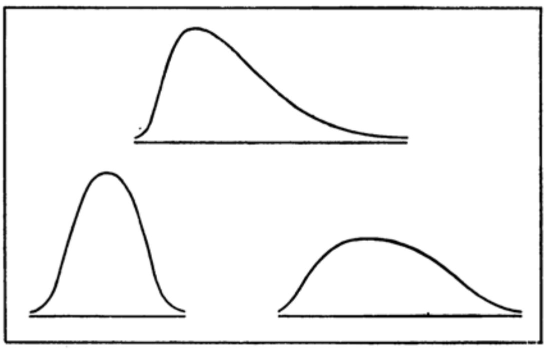


Fig. 184. Distributions associated with a natural pattern.

General

If the pattern on a control chart is in control for considerable periods of time, it means that we have a stable, steadily running process which is not being disturbed by outside causes. In a shop situation all that needs to be done in interpreting such a chart is to compare it with the specification limits or other authorized standards. In a capability study, however, this type of pattern may indicate that the engineering problem has not yet been solved. It is often necessary to disturb a natural pattern or a process running in control in order to bring about improvements, cost reductions, etc.

In dealing with a natural pattern, the engineer should keep in mind that causes not now identified are not necessarily unidentifiable. It is always possible to identify more of them if we are willing to exert the effort to do so. Theoretically it would be possible to keep on identifying and eliminating causes until all variability would be reduced to zero.

Practically, however, the causes become progressively more difficult to isolate or distinguish, so there is no practical possibility of reducing the variability to zero. The causes which are left in a so-called "natural pattern" are so small and balanced that it requires special effort (equivalent to setting up a new process) to reduce these causes further.

See page 150 for a discussion of "natural" or "non-assignable" causes.

F-16 STABLE FORMS OF MIXTURE

These are a special type of mixture, as explained on page 170. Stable mixtures result from the presence of more than one distribution, the distributions being in balance. The mixture is shown by a pattern which may or may not react to the formal tests but which indicates abnormality by the absence of the usual number of points near the center or edge of the chart. Stable mixtures will show up plainly on $\bar{X}$ charts, R charts, p-charts and charts for individual measurements. See Figures 185 and 186 on page 172.

The distribution consists of more or less widely separated components which do not change with respect to each other in either proportion or location. The samples may be taken from each distribution separately (in which case the mixture will show up on an $\bar{X}$ chart or p-chart), or they may be taken from the two distributions combined (in which case the mixture will show up on an R chart).

There are two forms of stable mixture which result from a special systematic way of taking the sample: these are called "Systematic Variable" (pages 175–176) and "Stratification" (pages 172–174).

When mixtures are stable, the causes producing the distributions are likely to be settled or permanent in nature: product coming steadily from two different sources, a difference in machine design, a consistent difference between first and second shift. Stable mixtures occur most frequently when measurements are taken on the end product instead of at the early operations. On the whole however, they are less common than unstable mixtures.

Typical causes which may produce stable mixtures are the following:

$\bar{X}$ Chart

Consistent differences in material, operators, etc., where the distributions are subsequently mixed.
Different lots of material in storeroom.
Large quantities of piece parts mixed on the line (in different pans or bins).
Code differences.
Differences in test sets or gages.

R Chart

Different lots of materials in storeroom.
Large quantities of piece parts mixed on the line (in same pan or bin).
Frequent drift or jumps in automatic controls.
Difference in test sets or gages.

p-Chart

Non-random sampling technique.
Lots coming from two or more different sources.
Screening of some lots at a prior operation.
Difference in process checkers.
Difference in test sets, gages etc.

Individuals Chart

Any of the causes which can affect the $\bar{X}$ chart or R chart.

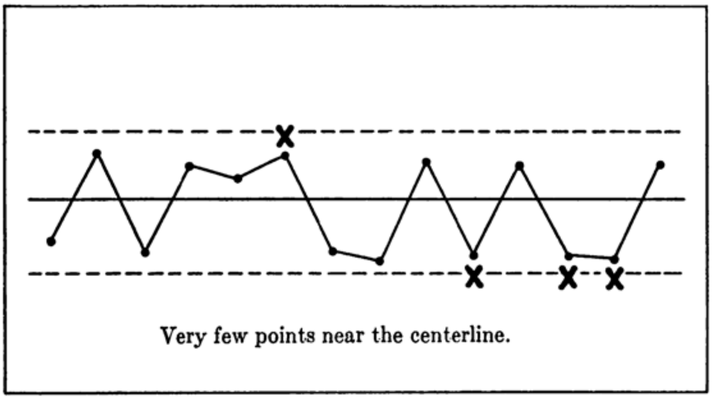


Fig. 185. Stable mixture. See also Figures 187 and 192.

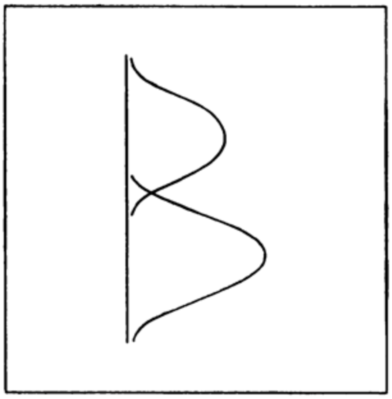


Fig. 186. Distribution associated with stable mixture.

F-17 STRATIFICATION

Stratification is a form of stable mixture which is characterized by an artificial constancy. Instead of fluctuating naturally inside the control limits, with occasional points approaching the upper and lower limits, a stratification pattern appears to hug the centerline with few deviations or excursions at any distance from the centerline. In other words, stratification is shown by unnaturally small fluctuations, or an absence of points near the edge of the chart.

As shown in Figure 188 the underlying distribution is a composite, made up of small distributions which are radically different.

We sometimes describe stratification by pointing out that the pattern is *unnaturally quiet*. Do not make the mistake of thinking that a pattern like this shows "good control." On the contrary, it shows lack of control because distributions that were intended to be the same are very different. The following is an explanation of the way in which a pattern of stratification forms.

Formation of a pattern of stratification

Stratification results when samples are taken consistently from widely different distributions, in such a way that one or more units in every sample will come from each of the distributions. The most common way of getting this effect is to allow the person who selects the sample to take one part from each operator in a group of operators, or one part from each machine, or each position on a machine, etc. Sometimes people do this without realizing the possible implications because they are anxious to make the sample "representative."

When sampling is done in this manner the selection of units is not random, and consequently the pattern will not fluctuate as in the case of randomness. On an R chart, for example, the *level* on the chart will be unnaturally high because of the separation between the distributions. The *fluctuations*, however, will be unnaturally small because the largest and smallest units in each sample are fairly similar. This can be seen in the following example.

> Imagine five machines which are turning out distributions very different from each other. See Figure 188. The spread of each individual distribution is ±.001, but the distance between the highest and lowest distributions is nearly .005. A process checker takes one part from each machine in making up a sample of 5.
>
> When the checker takes a sample in this manner and calculates the value of R, this value will consist mostly of the difference between the highest and lowest machines. Successive values of R will differ slightly from each other, but they will all be in the neighborhood of .005. Their pattern of fluctuation will be entirely different from a series of natural R values, which (if they had an average range of approximately .005) would fluctuate all the way from zero to nearly .011. In Figure 188 there is no possibility of getting a range higher than .007 or less than .003. Consequently the

pattern will show unnaturally small fluctuations as compared with a natural pattern having the same average range.

Similar reasoning will show how such a pattern forms on other types of chart.

By a careful study of the stratified pattern on an R chart, it is possible to estimate how far the distributions are separated.

Stratification on a p-chart

Stratification patterns may form on a p-chart if there are large differences between various containers of product and if the samples are always selected in such a way as to include some units from each container. An extreme example is the following:

If one container were composed solely of defectives and there were no defectives in any of the other containers, and if the inspector took an equal number of units from each container, all the inspector's samples would contain exactly the same number of defectives. A p-chart on the inspector's data would show nothing but a straight line at the average percent defective. The stratification in this case would be so great that it would remove all sampling fluctuation.

Causes of stratification

The cause of stratification may be any element in the process which is consistently being spread across the samples. It will probably be the machine if you are taking one part from each machine. It will probably be the spindle if you are taking one part from each spindle. It will be the boxes of product if you are taking part of your sample from each box. Among the most common causes for stratification are the following:

$\bar{X}$ Chart

Anything which is capable of causing mixture may also produce stratification. However, stratification shows up less readily on the $\bar{X}$ chart than on the R chart.

Apparently stratified patterns on an $\bar{X}$ chart are frequently the result of incorrect calculation of the control limits.

The misplacing of a decimal point may cause an apparent effect of stratification.

R Chart

Any of the causes listed under Stable Mixture.

p-Chart

Any of the causes listed under Stable Mixture.

Individuals Chart

Since true stratification results from spreading a sample across two or more distributions, this type of pattern cannot occur on a chart for individual measurements.

Sometimes, however, the control limits on a chart for individuals may become inflated by

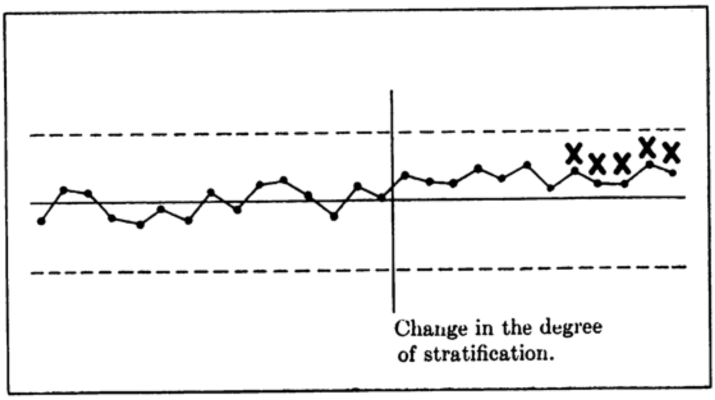


Fig. 187. Stratification.

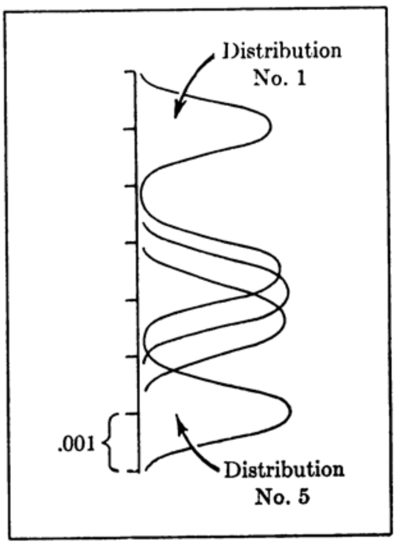


Fig. 188. Distribution associated with stratification.

erratic data or mixtures. This may in certain cases produce an effect similar to stratification.

F-18 SUDDEN SHIFT IN LEVEL

A sudden shift in level is shown by a positive change in one direction. A number of x's appear on one side of the chart only.

If the two periods are plotted separately in a frequency tabulation, the underlying distributions will be separate and distinct. If the two periods are combined, the distribution may be wide or show separate peaks. Sudden shifts may show up on any of the commonly used control charts.

(1) On an $\bar{X}$ chart this type of pattern indicates the sudden introduction into the process of a new element or cause (usually a simple or single cause) which moves the center of the distribution to a new location and then ceases to act on it further. The pattern shifts up or down from the centerline and rapidly establishes itself around the new level.

(2) On an R chart a sudden rise in level generally indicates the introduction of a new distribution in addition to the distribution previously in the product. A sudden drop in level generally indicates that one or more distributions have been removed.

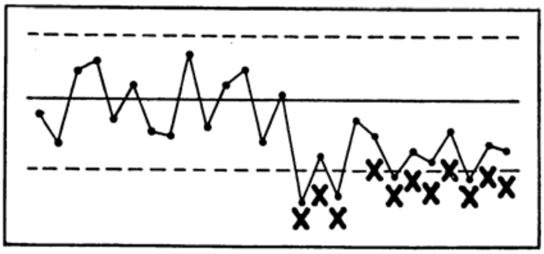


Fig. 189. Sudden shift in level.

(3) On a p-chart this type of pattern indicates a major change in the distribution of product or in the method of measuring the product. As indicated below, the p-chart is interpreted differently depending on whether the change is in the high direction or the low.

(4) On a chart for individuals a shift in level is interpreted the same way as a similar shift on the $\bar{X}$ chart.

A sudden shift in level is one of the easiest patterns to interpret on any chart. Typical causes include the following:

$\bar{X}$ Chart

(R chart must be in control.)

Change to a different kind of material.
New operator.
New inspector.
New test set.
New machine.
New machine setting.
Change in set-up or method.

R Chart

Change in motivation of operators.
New operators.
New equipment.
Change to different material or different supplier of piece parts.

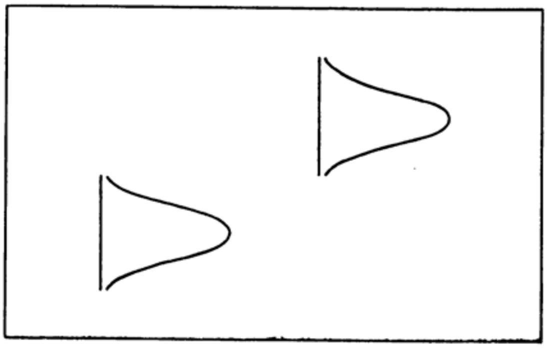


Fig. 190 Distribution associated with sudden shift in level (separate plotting).

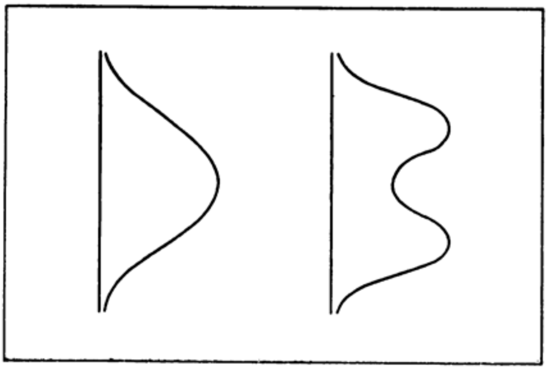


Fig. 191. Distribution associated with sudden shift in level (combined plotting).

The following causes will make the R pattern rise:

Greater carelessness on the part of the operators.

Inadequate maintenance.

Less expensive or less accurately designed machines and facilities.

Positioning or holding device in need of repair.

Anything which tends to make the product less uniform.

The following causes will make the R pattern drop:

Improved workmanship.

Machines or facilities with better capability.

Anything which tends to increase uniformity.

p-Chart

Changes in level are commonly due to:

New lot of material.

Change from one machine or operator to another.

Change in the calibration of a test set.

Change in method.

Change in standards.

Higher level on p-chart indicates:

Worse material.

Poorer machines, tools, fixtures, piece parts etc.

New or less adequately trained operators.

Tightening or addition of requirements.

Lower level on p-chart indicates:

Better operators.

Better machines, tools, piece parts etc.

Better methods or materials.

Loosening or removal of requirements.

Individuals Chart

Any of the causes which affect $\bar{X}$ charts, R charts or p-charts.

F-19 SYSTEMATIC VARIABLES

One of the characteristics of a natural pattern is that the point-to-point fluctuations are unsystematic or unpredictable. If the pattern for any reason becomes predictable (for example, if a low point is always followed by a high one or vice versa) the pattern is not natural and there must be an assignable cause. A systematic pattern of any kind indicates the presence of a systematic variable in either the process or the data. The most common appearance of such a pattern is a regular sawtooth effect like that in Figure 192 on page 176.

The distribution which accompanies this pattern is wide and flat-topped. It may or may not show separate peaks, depending on the distance between the high and low points.

Cycles are one form of systematic or repeating pattern. (See pages 161–162.) Systematic variables may originate in either the process or the data.

Systematic variables in the process

Any of the causes listed under cycles on the $\bar{X}$ chart may act as systematic variables if they alternate on a regular basis. For example, day shift always high, night shift always low.

Systematic variables in the data

These are often introduced by the way in which the data are divided in forming samples. For example, an engineer was testing the same units repeatedly over a period of time in order to study possible deterioration or drifting. Ten such units were being tested but as a matter of convenience he wished to plot samples of 5. He divided the 10 units into two groups of 5 and plotted them alternately on the chart. The result looked very much like the pattern in Figure 192.

The systematic ups and downs in such a case are not due primarily to process changes (which the chart is intended to analyze) but rather to the fact that one group of units happens to have a higher average (or range) than the other. The difference between groups may be so large that it will not be possible to detect other variation. The best way to avoid the systematic effect is to plot two separate charts, one for each group of units.

Among the causes for systematic variation are the following:

$\bar{X}$ Chart

Difference between shifts.

Difference between test sets.

Difference between assembly lines where product is sampled in rotation.

Systematic manner of dividing the data.

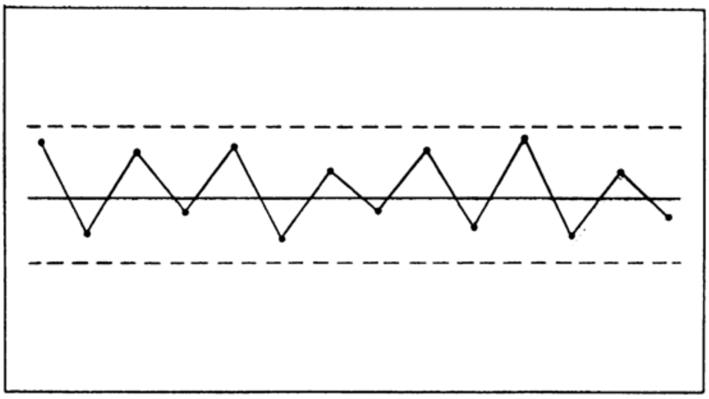


Fig. 192. Systematic variable.

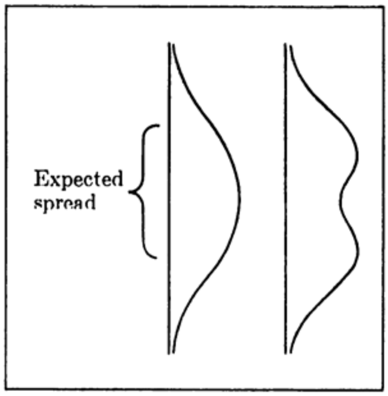


Fig. 193. Distribution associated with systematic variable.

R Chart

This effect is generally due to a systematic manner of dividing the data.

Less frequently there may be a large difference in spread between different conveyors, shifts, sources of material, etc., being sampled in rotation.

p-Chart

This effect is almost always due to drawing the samples systematically from different sources.

Individuals Chart

Systematic variation is often due to differences between tools, chucks, positions, assembly fixtures, locating holes, etc.

It shows up particularly when the measurements are being recorded in the order of production, and when the above elements in the process are contributing in succession to the production order.

F-20 TENDENCY OF ONE CHART TO FOLLOW ANOTHER

Part of the meaning of randomness is that the pattern is unpredictable and never repeats. Two control charts which are individually fluctuating at random and which are not in any way connected with each other in a cause and effect relationship will have no tendency to follow each other. Conversely, when two control charts do follow each other it indicates at least the possibility of some relationship between them.

There are two ways in which patterns may tend to follow each other.

(1) There may be a point-to-point correspondence. That is, the individual points may tend to move up and down in unison with respect to other nearby points. See Figure 194. If this happens regularly over a long series of points it indicates some relationship between them.

(2) There may be a level-to-level correspondence. That is, the two patterns may tend to show shifts in level at the same time or to follow trends simultaneously. See Figure 195. This may or may not be accompanied by a point-to-point correspondence also.

Point-to-point correspondence

Point-to-point correspondence generally occurs when the samples plotted on the two corresponding patterns were the same samples. For example, the corresponding $\bar{X}$ and R points on the same control chart are obtained from the same samples. It is also possible for the points on different charts to come from the same samples. For example, we may take a

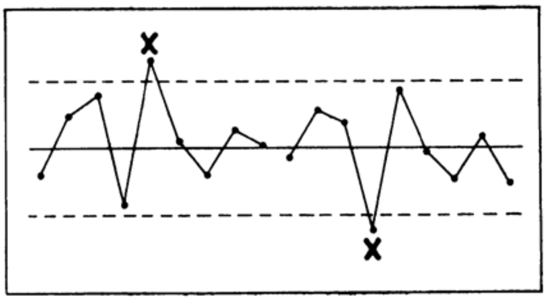


Fig. 194. Point-to-point correlation.

sample of 5 parts and measure these parts for several different characteristics which are then plotted on separate control charts but represent the same samples.

When point-to-point correspondence is observed between the two patterns on the same $\bar{X}$ and R chart (that is, when the $\bar{X}$ chart tends to follow the R chart, point by point) this indicates skewness in the underlying parent distribution. See page 156. When point-to-point correspondence is observed between two different characteristics on two different $\bar{X}$ and R charts (for example, when the $\bar{X}$ points for Power Output tend to go up or down with the $\bar{X}$ points for a certain dimension "B") it indicates that there is probably a cause and effect relationship between the two characteristics. The cause and effect relationship may be a direct one (Dimension "B" actually causes a change in Power Output) or it may be indirect through a third variable (Dimension "B" is affected by a certain spacing and this spacing also governs Power Output).

When two characteristics show point-to-point correspondence on their $\bar{X}$ charts over a considerable period of time, the indicated relationship is very close. When the two R charts in addition show point-to-point correspondence, the relationship is even stronger. If two characteristics have practically a 1-to-1 relationship (and the same samples are used for both charts) their patterns will be almost duplicates. $\bar{X}$ and R charts can be used in this manner to study correlation.

Level-to-level correspondence

When a level-to-level correspondence is observed between the two patterns on the same $\bar{X}$ and R chart (that is, the $\bar{X}$ level changes at the same time as the R level), this does not indicate any necessary relationship. Except in rare cases, where the standard deviation of a characteristic happens to be proportional to its magnitude, the $\bar{X}$ and R levels are completely independent.

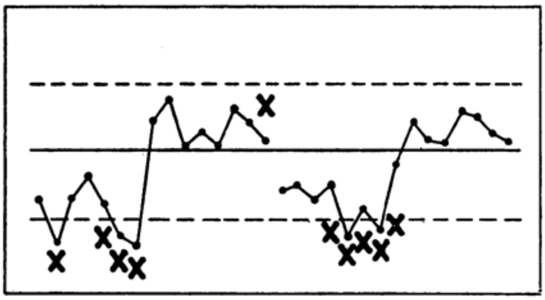


Fig. 195. Level-to-level correlation.

When a level-to-level correspondence is observed between two different characteristics on two different control charts (whether or not they come from the same samples) it may or may not indicate that the two characteristics are related. For example, many p-charts tend to show improvement in the early stages of a process control program. While these are undoubtedly tied together loosely by a common cause—the increased emphasis on process control—there is no reason to suppose that one characteristic causes or governs another.

While it cannot be said that a level-to-level correspondence necessarily indicates a cause and effect relationship, on the other hand it is definitely true that wherever a cause and effect relationship exists the patterns will tend to react together. Consequently, when the engineer observes the levels changing together on various charts, he should at least check to satisfy himself whether a relationship exists. In making such a check he should first examine his theoretical knowledge to see whether such a relationship would be reasonable; remembering, of course, that new knowledge comes from the discovery of relationships which were not suspected before. If it appears reasonable that the two characteristics might be in fact related, the engineer should check them further, following the methods suggested on page 56.

F-21 TRENDS

A trend is defined as continuous movement up or down; x's on one side of the chart followed by x's on the other; a long series of points without a change of direction. Two examples of trends are shown on page 30.

When a trend is present, the total distribution is flat-topped and wider than would be predicted by an R chart. It shifts its location gradually in one direction over a period of time. Trends are in general fairly easy to identify and associate with the process.

Trends may result from any causes which work on the process gradually. The nature of the cause can be determined by the type of

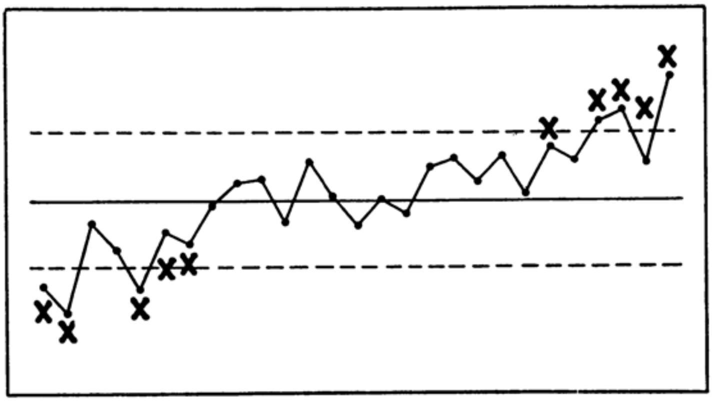


Fig. 196. Trend.

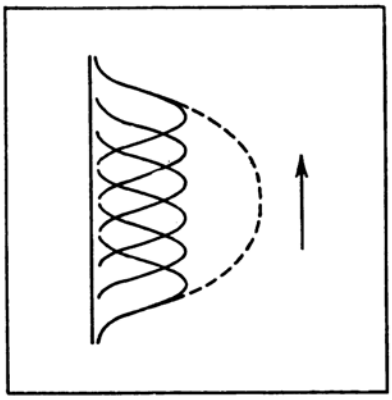


Fig. 197. Distribution associated with a trend.

chart on which the trend appears. If it appears on the $\bar{X}$ chart, the cause is one which moves the center of the distribution rather steadily from high to low or vice versa. If it appears on the R chart, it is gradually increasing or decreasing the spread. If it appears on a p-chart, it is gradually increasing or decreasing the percent defective, etc. Caution needs to be used in the interpretation of trends because it is very easy to think we see trends where none really exist. The irregular up-and-down fluctuations in a natural pattern often appear to the uninitiated to look like trends.

Sometimes the changes which show up as trends on a chart are not really gradual. Sudden changes in the process frequently look like trends either as a result of shop practices which prevent a sharp cut-off or change-over, or merely as a result of chance fluctuations in the data. Some of the most frequent causes of trends are the following:

$\bar{X}$ *Chart*

(*R* chart must be in control.)

Tool wear.
Wear of threads, holding devices or gages.
Deterioration of plating or etching solution.
Aging.
Inadequate maintenance on test set.
Seasonal effects, including temperature and humidity.
Human variables. (These may be affected by the amount of supervisory attention, etc.)
Operator fatigue.
Increases or decreases in production schedules.
Gradual change in standards.
Gradual change in proportions of lots.
Poor maintenance or housekeeping procedures. For example, accumulation of dirt or shavings, clogging up of fixtures or holes.
Pumps becoming dirty.
Degreaser becoming exhausted.

R Chart

Increasing trend

Something loosening or wearing gradually.
Dulling of a tool.
Change in proportion of lots.
Various types of mixture.

Decreasing trend

Gradual improvement in operator technique.
Effect of better maintenance program.
Effect of process controls in other areas.
Product more homogeneous, or less affected by mixture.

p-Chart

Trend upward.

This means the process is turning out more defectives. The trend may be due to:
- Introduction of poorer material.
- Poorer work by operators.
- Tool wearing too far.
- Drift in a test set.
- Tightening or addition of requirements.

Trend downward.

This means the process is turning out fewer defectives. The trend may be due to:
- Increasing skill or greater care on the part of the operators.

Better material or tools for the operators to work with.

Relaxation of requirements.

Relaxation of standards.

Individuals Chart

Anything which causes trends on the $\bar{X}$ chart, and to a lesser extent the R chart, may affect a chart of individuals. Indications of trends are less reliable, however, on the chart for individuals, and should ordinarily be checked with an $\bar{X}$ and R chart.

F-22 UNSTABLE FORMS OF MIXTURE

These are a special type of mixture, as explained on page 170. Unstable mixtures are one of the most common types of pattern and also one of the most important. This type of mixture is caused by having several distributions in the product which are capable of shifting or changing with respect to each other. The change may be a change in location or merely a change in proportion. For example: one distribution may be just coming into the process or just going out; one of the distributions may be shifting its average or spread with respect to some of the others.

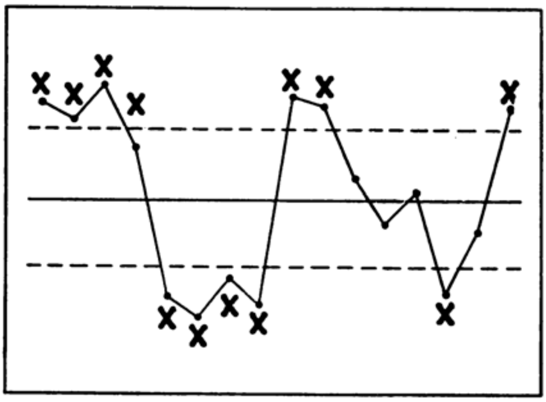


Fig. 198. Unstable mixtures.

The mixture pattern appears on the $\bar{X}$ chart when samples are taken separately from the different sources of product, and on the R chart when samples are taken at random from the different sources combined. In either case, unstable mixtures tend to show up quite plainly on a p-chart or chart for individual measurements.

Unstable mixtures are closely related to four other types of pattern:

1. Instability.
2. Interaction on the R chart.
3. Grouping or bunching.
4. Freaks.

In general, the detection and elimination of unstable mixtures will tend to make other patterns easier to interpret.

Since the various forms of mixture are largely a matter of degree, any of the unstable patterns may change quite rapidly from one type to another. For example, if freaks or wild readings in the data become fairly numerous, they will be interpreted as unstable mixture. Among the common causes for unstable mixtures are those listed on the following page.

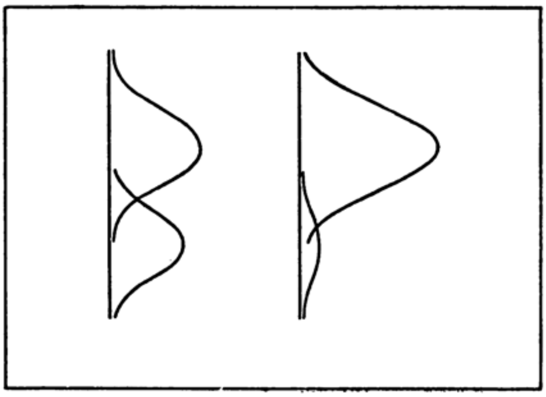


Fig. 199. Distribution associated with unstable mixture (change in proportion).

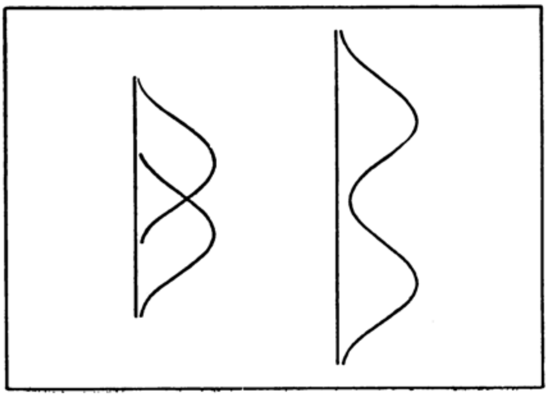


Fig. 200. Distribution associated with unstable mixture (change in location).

$\bar{X}$ Chart

(R chart must be in control.)

Distribution changing due to differences in material, operators, test sets, etc.
Breakdown in facilities or automatic controls.
Overadjustment of the process.
Effect of experimental or development work.
Carelessness in setting temperature control, timing device, etc.
Wrong sampling procedures.
Change in the method of measurement.
Errors in plotting.
Incomplete operation.
Setup parts.

R Chart

Two or more materials, machines, operators, machine setters, test sets, gages, etc.
Too much play in a fixture.
Holding or locking devices unreliable.
Mixture of material.
Looseness of a chuck.
Maintenance schedules not adequate.
Operator in need of further training.
Operator fatigue.
Tool in need of sharpening.
Machine in need of repair.
Fixtures or holders not holding the work in position.
Lack of alignment, etc.
Accidental damage.
Operation not completed.
Breakdown of facilities.
Unstable testing equipment.
Experimental units.
Defective piece parts.
Error in calculating or plotting.

p-Chart

Serious lack of control in the process producing a series of lots.
Use of unreliable checking equipment or methods.
Characteristics which tend to be "all good" or "all bad."
Unsystematic screening by shop prior to the time when the product reaches the checking point.
Variations in sample size.
Non-random sampling.

Individuals Chart

Any of the causes capable of affecting the $\bar{X}$ chart or R chart.

F-23 CALCULATION OF TESTS FOR UNNATURAL PATTERNS

Tests for unnatural patterns are obtained by simple probability calculations as shown below. It is necessary to know the probability associated with the portion of the control chart to which the test will apply. For $\bar{X}$ and R charts, use the probabilities given in Figures 201 and 202. For other control charts, use (a) the $\bar{X}$ and R tests or (b) special tests calculated as on page 183.

In all cases, the standard three sigma limit should be included as one of the tests.

Tests for $\bar{X}$ chart

In the case of an $\bar{X}$ chart, we assume that the distribution is approximately normal. The tests are calculated in such a way that if the process is in control, and if the tests are applied simultaneously to one-half of the control chart at a time (upper half or lower half), the probability of getting a reaction to the tests will be about .01.

The following tests can be calculated using the probabilities in Figure 201.

(1) Consider only the outer third in the upper half of the chart. The probability of getting one point at random in this area or beyond is .0227. The probability of getting two points in succession in this area or beyond is .0227 x .0227 or .00052.

Since this is a smaller probability than we wish to use for the test, we increase the probability by letting the test apply to two out of three successive points instead of two points in succession. The probability of "two out of three" is calculated as follows:

The probability of getting two points in succession in the outer third of the chart or beyond is .0227 x .0227. The probability of getting a third point in the sequence in some other portion of the chart (but not in this particular outer third or beyond) is

NORMAL DISTRIBUTION

	Probability = .00135	
		3 σ Control Limit
Outer third	Probability = .02135	
Middle third	Probability = .1360	
Inner third	Probability = .3413	
		Centerline
Inner third	Probability = .3413	
Middle third	Probability = .1360	
Outer third	Probability = .02135	
		3 σ Control Limit
	Probability = .00135	

Fig. 201. Probabilities used in tests for unnatural patterns: $\bar{X}$ charts.

DISTRIBUTION OF RANGES WHEN THE PARENT POPULATION IS NORMAL

		Samples of 5	*Samples of 2*	
	Probability =	.0046	.0094	
				3 σ Control Limit
Outer third	Probability =	.0294	.0360	
Middle third	Probability =	.1231	.1162	
Inner third	Probability =	.3120	.2622	
				Centerline
Inner third	Probability =	.3396	.1724	
Middle third	Probability =	.1735	.1910	
Outer third	Probability =	.0178	.2128	
				3 σ Control Limit
	Probability =	.0000	.0000	

Fig. 202. Probabilities used in tests for unnatural patterns: R charts.

1 − .0227 or .9773. The probability of getting a run of three points, two of which are in this particular outer third (or beyond) and one of which is not, is .0227 x .0227 x .9773 x 3 = .0015. The probabilities are multiplied by 3 because the odd point in the series—the one which does not count in the test—might be either the first, the middle or the last in the series.

(2) Consider now all points in the middle third or beyond. The probability of getting a single point at random in this area is .0227 + .1360, or .1587. The probability of getting four points in a series in this area and one point in some other area is .1587 x .1587 x .1587 x .1587 x .8413 x 5 = .0027.

(3) Consider now all the points on one side of the centerline. The probability of getting one such point at random is .3413 + .1360 + .0227, or .50. The probability of getting eight points in succession on one side of the centerline is .50 x .50 x .50 x .50 x .50 x .50 x .50 x .50 = .0039.

These three probabilities, when added to the probability of exceeding the regular three sigma limit, give a total of .0094, as shown on page 183.

The actual probability associated with the combined tests is somewhat less than this total since there is a certain probability that the same point may react to more than one test.

All the above tests are "one-sided" tests. That is, they are calculated from the probabilities on one side of the centerline only. Tests for stratification and mixture, on the other hand, are usually calculated from the "two-sided" probabilities. It is possible to calculate any desired number of tests by using the principles illustrated above. Examples of this will be found in Reference No. 20.

Tests for R chart

The distribution of ranges is not normal even when the parent population is normal. It varies according to the size of the sample from which the range is computed. The probabilities associated with ranges of samples of 5 and ranges of samples of 2 are shown in Figure 202. Note that these probabilities are quite different from those of the normal distribution.

For ranges of samples of 5, however, it is possible to use the same tests as for a normal distribution without producing a large difference in the total reaction to the tests. This is shown on page 183. Note that the probability of reaction may be quite different in the case of individual tests, but the sum of these probabilities is not far from .01.

The $\bar{X}$ tests may therefore be adopted as "standard" tests and used not only for the $\bar{X}$ chart but also for the R chart when the sample size is 4 or 5. This is a practical advantage in many routine applications of control charts, since it is not necessary to learn more than one set of tests.

Samples of 2

For ranges of samples of 2, the probabilities are sufficiently different to make it advisable to use a separate set of tests. Suitable tests are shown in Figure 203.

If the parent population is normal, the probabilities associated with the tests in Figure 203 are as follows:

Upper Half of Chart	*Ranges of Samples of 2*
Single point out	.0094
2 successive points	.0021
3 successive points	.0042
7 successive points	.0025
Total	.0182

Lower Half of Chart	*Ranges of Samples of 2*
10 successive points	.0040
6 successive points	.0043
4 successive points	.0020
Single point out	—
Total	.0103

Even with the special tests, note that the total of all tests for the upper half of the chart approaches the .02 probability rather than .01. This is because of the large probability of getting a single point outside of the standard three sigma limit. The latter probability itself is nearly .01.

Ranges of samples of 2 are used mainly in Designed Experiments, where there is no need

Upper Half		*Lower Half*	
Single point out			
A	2 succ. points in Zone A or above		
B	3 succ. points in Zone B or above		
C	7 succ. points in Zone C or above		
		C	10 succ. points in Zone C or below
		B	6 succ. points in Zone B or below
		A	4 succ. points in Zone A or below
		—	Point out is not possible

Fig. 203. Tests applied to the R chart when $n = 2$.

to avoid the complication of using a second set of tests.

Tests for p-charts and various other control charts

On most charts where control limits are reasonably symmetrical, it is sufficiently accurate to use the standard tests. However, by making use of the principles described above, it is possible to calculate special tests for *p*-charts or other charts whose control limits may at times be unsymmetrical. First find the probabilities associated theoretically with each third of the control band, and calculate tests which will result in the desired total probability of getting a reaction to the tests.

For a *p*-chart use either Binomial or Poisson probabilities: for a *c*-chart use the Poisson, etc.

The tests in the following column are roughly equivalent to each other when applied to areas having the indicated probabilities, and can be used as a general guide. The probability of reaction to each test is approximately .0014 ± .0002.

Total Probability Derived from Distribution (in a particular zone and beyond)	*Suitable Test*
.02	2 out of 3
.04	2 successive points
.11	3 successive points
.13	4 out of 5
.20	4 successive points
.27	5 successive points
.33	6 successive points
.40	7 successive points
.44	8 successive points
.48	9 successive points
.52	10 successive points

Practical shop applications

In most cases, when doing practical work in the shop, it is sufficient to use the tests explained on pages 23–28. If special tests are wanted to fit a particular situation, the tests should be calculated and checked by the responsible Quality Control Team.

Standard Control Chart Tests
(as given on pages 23–28)

	Probability of Getting a Reaction to the Tests	
Upper Half of Chart	*Normal* ($\bar{X}$)	*Ranges of Samples of 5*
Single point out	.0013	.0046
2 out of 3	.0015	.0033
4 out of 5	.0027	.0026
8 in a row	.0039	.0023
Total	.0094	.0128
Lower Half of Chart	*Normal* ($\bar{X}$)	*Ranges of Samples of 5*
8 in a row	.0039	.0063
4 out of 5	.0027	.0054
2 out of 3	.0015	.0009
Single point out	.0013	—
Total	.0094	.0126