



Section I

Fundamental Principles

PART A

Introduction to Statistical Quality Control

Statistical Quality Control is a scientific method of analyzing data and using the analysis to solve practical problems. It can be applied to anything that it is possible to express in the form of numbers. In a manufacturing plant it can be applied to

- Engineering problems
- Operating problems
- Inspection problems
- Management problems
- Accounting and clerical problems

or virtually any other field of activity. Few of us can think of anything, in connection with our own jobs, that is not associated in some way with numbers.

A-1 MEANING OF THE TERM “STATISTICAL QUALITY CONTROL”

The word “Statistical” means “having to do with numbers,” or more particularly with “drawing conclusions from numbers.”

The word “Quality” means much more than the goodness or badness of a product. It refers to the *qualities* or *characteristics* of the thing or process being studied.

The word “Control” means “to keep something within boundaries,” or “to make something behave the way we want it to behave.”

Taken together, and applied to a manufacturing operation, the words **Statistical Quality Control** mean this:

STATISTICAL	With the help of numbers, or data,
QUALITY	We study the characteristics of our process
CONTROL	In order to make it behave the way we want it to behave.

The term “process,” as used in the previous statement, is capable of assuming many different meanings.

A-2 MEANING OF “PROCESS”

A “process” is any set of conditions, or set of causes, which work together to produce a given result. In a manufacturing plant we usually think first of a series of fabricating and assembling operations which result, for example, in the production of cable, electron tubes, relays, switchboards and other apparatus or equipment.

However, since the word “process” means merely a *system of causes*, a process may be far simpler than the ones mentioned above, or it may be far more complex. In statistical quality control the “process” we choose to study may be:

- a single machine, or a single fixture or element of a machine.
- a single human being, or a single motion performed by a human being.
- a piece of test equipment.
- a method of measurement or gaging.
- a method of assembly.
- the act of typing (or performing any clerical operation).
- a group of many machines turning out different or similar pieces of product.
- a group of many human beings (for example, a pay group or a Shop).
- a combination of human beings, machines, materials, methods, pieces of equipment, etc. For example, the procedures required to manufacture

- switches, rubber or wire.
- a method of processing, such as chemical treatment or plating.
 - a mental activity, such as visual checking or making calculations.
 - intangible human elements, such as attitudes, motives and skills.
 - the whole mass of causes which result in a Guided Missile System (for example), including thousands of resistors, capacitors, tubes and other components furnished by hundreds or thousands of different suppliers.
 - anything else which results in a series of numbers and is connected with unsolved problems.

In its narrowest sense the term "process" refers to the operation of a single cause. In its broadest sense it may refer to the operation of a very complicated "cause system." This is why it is possible to make "process capability studies" in connection with practically any type of Engineering, Operating, Inspection or Management problem; including such problems as overall merchandise losses, the overall cost of maintenance or plant-wide inspection ratios.

A-3 ESSENTIAL TECHNIQUES IN STATISTICAL QUALITY CONTROL

The following techniques are essential in the quality control programs to be described in this Handbook:

- (1) Process Capability Studies (the basic use of control charts).
- (2) Process Control Charts (the use of control charts in the shop).
- (3) Statistical Sampling Inspection (which may or may not make use of control charts).
- (4) Statistical Design of Experiment (a device for comparing variables and determining their significance. This also is frequently handled with control charts).

The principal concept presented in this Handbook is that of the control chart, which consists of a plotted series of observations or "samples." The unique feature of the control chart is its ability to form data into patterns which, when tested statistically, can lead to information about the process.

The Process Capability Study is used by engineers, Operating supervisors and management to obtain information about how a process behaves.

Process Control Charts are used in the shop to increase yields, cut down scrap and repair, and help the people to do better work.

Statistical Sampling Inspection is used by inspectors as a scientific basis for accepting or rejecting product.

Design of Experiment is used in research and development problems, to study the effect of many variables at once, to throw light on mysterious or puzzling causes and effects, and to explore the unknown.

These four techniques are described more fully in later parts of the Handbook.

PART B

Introduction to Control Charts

B-1 STATISTICAL PHENOMENA IN THE WORLD AROUND US

B-1.1 Fluctuating patterns

If data are collected which have a bearing on any problem, any series of events or any manufacturing situation, these data are always found to exhibit variation. Instead of being exactly the same from point to point or from time to time, the numbers vary. If plotted on a piece of graph paper, so the variations can be studied, the numbers always form a fluctuating, zig-zag pattern. Some typical examples of fluctuating patterns are shown in Figures 1-4.

In the first case an operator was assembling spoolheads onto a core. The supervisor kept a record of the percentage of loose spoolhead assemblies made by this operator day after day. He found that the percentages varied. The percentage of loose spoolheads was not the same every day.

Figure 2 is a record obtained from an automatic welding operation. This is an entirely different type of operation from Figure 1. It involves different materials, different facilities, a different operator, different problems. When the supervisor kept a record of the daily percentage of offcenter welds, he found that the percentages varied in much the same manner as Figure 1. The percentage of offcenter welds was not the same every day.

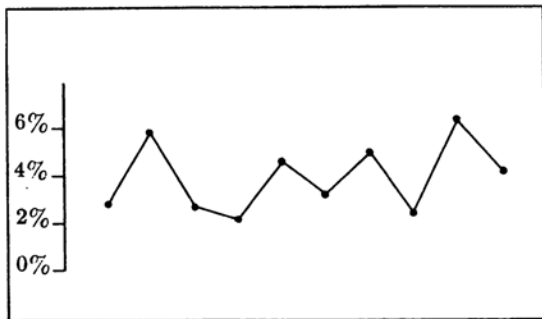


Fig. 1. Daily percentage of loose spoolheads.

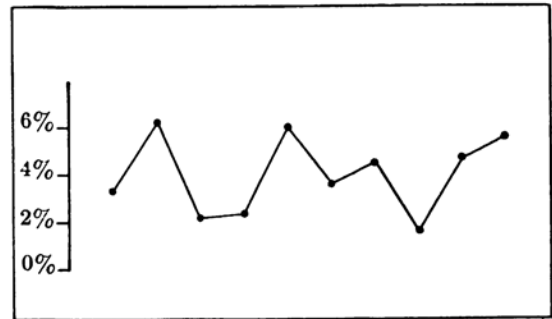


Fig. 2. Daily percentage of offcenter welds.

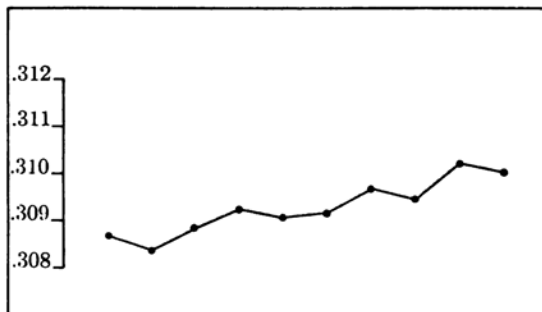


Fig. 3. Successive parts coming from a screw machine (diameter).

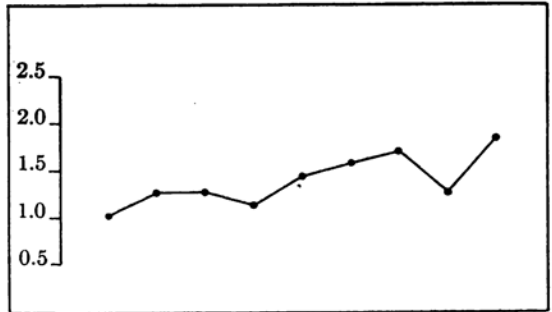


Fig. 4. Electrical measurement on a series of assemblies (noise level).

The same thing was found on plotting the dimensions of successive parts coming from a screw machine (Figure 3), and also the electrical characteristics of a series of assembled units (Figure 4). Any series of numbers from a process, if plotted in sequence, will form a fluctuating pattern. Even repeat measurements made on the same object will not be exactly the same, time after time or day after day.

Similar variation is found in accounting figures, production figures, records of attendance, temperatures, pressures, medical reports or any other set of numbers from an industrial process. We do not know of any type of manufacture in which variation is not present.

B-1.2 What causes the fluctuations in a fluctuating pattern?

Fluctuations in the data are caused by a large number of minute variations or differences: differences in materials, equipment, the surrounding atmospheric conditions, the physical and mental reactions of people. Most of these differences are extremely small. They cause the pattern to fluctuate in what is known as a "natural" or "normal" manner. Occasionally, however, there will be a large or unusual difference, much more important than all the other differences put together. For example, material is taken from a different batch; the machine setter makes a new setting; an inexperienced operator takes the place of an experienced operator. These large causes make the pattern fluctuate in an "unnatural" or "abnormal" manner.

Experience shows that there are definite detectable differences between the "natural" and "unnatural" patterns. It is possible to discover and study these differences by means of simple calculations based on well-known statistical laws.

Once we know that a pattern is unnatural, it is possible to go further and find the *cause* of the unnaturalness. This makes it possible to detect, identify and study the behavior of *causes*.

B-1.3 Distributions

Fluctuations are not the only statistical phenomena which are observable in nature.

It has long been known that if we take large numbers of observations on some physical quantity (such as the charge on an electron), or large numbers of measurements on an industrial product (such as the diameter of a wire or shaft), these measurements will tend to group themselves around some central value with a certain amount of variation or "scatter" on either side. The pattern or shape formed by the grouped measurements is called a "frequency distribution." We observe that if the causes which produce the observations or measurements remain essentially unchanged, the distribution tends to have certain distinguishable and stable characteristics. These characteristics become more definite as the number of observations or measurements increases. We conclude from this that, if the cause system is constant, the observed distribution tends to approach, as a statistical limit, some distribution function or "law."

This tendency to form a distribution is observed throughout nature. It is one of the most fundamental of all natural laws.

Experience tells us that the two sets of statistical phenomena—distributions and fluctuations—are not separate and unrelated. A distribution can be thought of as a *composite mass of fluctuations*, and the fluctuations can be thought of as *confined within the limits of a distribution*. It is therefore possible to make use of statistical limits, derived from distributions, to predict the behavior of a fluctuating pattern *when there are no abnormal causes*.

This can be stated formally as follows:

Whenever we have a series of events proceeding from a given system of causes, those events will not in general be identical with each other. Instead, they will fluctuate or vary in a manner described as "random." Nevertheless, if the cause system remains unchanged, the frequencies with which the events occur will tend to approach an objective probability, or set of probabilities, as the number of events increases indefinitely.

Translated into industrial terms, this can be stated as follows:

Whenever we have a series of observations or measurements, obtained from a given

process, those measurements will not in general be identical with each other. Instead they will vary in such a way as to form a fluctuating pattern. Nevertheless, if nothing disturbs the process, these fluctuating measurements will be held within definite mathematical limits. In the aggregate, a large number of these measurements will tend to form a predictable distribution.

Translated into everyday language, the statements above mean this:

- a. Everything varies.
- b. Individual things are unpredictable.
- c. Groups of things from a constant system of causes tend to be predictable.

Check your understanding of these fundamental concepts by studying the following simple examples:

Example 1

- a. People live to different ages.
- b. No one knows how long he himself will live.
- c. Insurance companies can tell with great accuracy what percentage of people will live to be 60, 65, 70 etc.

Example 2

- a. You cannot write the letter "a" twice in exactly the same way.
- b. You have no way of knowing how your next "a" will differ from the last one.
- c. Nevertheless there is something about your "a's" that makes them recognizably different from my "a's."

Example 3

- a. All patterns fluctuate.
- b. The individual points are unpredictable.
- c. A group or series of points from a constant process will tend to follow a pattern that obeys a fixed law.

These concepts are carefully developed by Shewhart in Reference No. 37.

B-1.4 Statistical limits for fluctuating patterns

By making use of certain equations, derived from statistical laws, it is possible to calculate "limits" for any given pattern. If a pattern is natural, its fluctuations will fit within these limits. If a pattern is unnatural, its fluctuations will not fit these limits. The following are examples of the calculation of statistical limits.

- (1) *Statistical limits for the chart on the spoolhead operation (Figure 1).*

To calculate limits for this chart, proceed as follows:

- a. First take the percentage of loose spoolheads turned out by the operator, on the average, over a period of time. In this case it was 4%.

The average percentage is called \bar{p} (pronounced *p*-bar). The "*p*" means percentage or proportion, and the bar above it means average.

- b. Then take the average number of spoolheads assembled by this operator during the day. In the example used, this number was 400.

The number assembled is called *n* (meaning number).

- c. The equation for the calculation of statistical limits is as follows:

$$\begin{aligned} \text{Limits of fluctuation} &= \pm 3 \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} \\ &= \pm 3 \sqrt{\frac{.04 (.96)}{400}} \\ &= \pm 3 \times .0098 \\ &= \pm .0294 \end{aligned}$$

The limits of fluctuation are therefore 2.94% on either side of the average, or 6.94% and 1.06%. The pattern for the spoolhead process should stay inside of these limits.

- (2) *Statistical limits for the chart on the welding operation (Figure 2).*

To calculate limits for this chart, proceed as follows:

- a. Use the same equation that was used in (1) above.
- b. The average percentage of offcenter welds (\bar{p}) was again 4%.

c. The average number of welds made by the operator per day (n) was in this case 1000.

$$\begin{aligned}
 \text{d. Limits of fluctuation} &= \pm 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \\
 &= \pm 3 \sqrt{\frac{.04(.96)}{1000}} \\
 &= \pm 3 \times .0062 \\
 &= \pm .0186
 \end{aligned}$$

The limits of fluctuation are therefore 1.86% on either side of the average, or 5.86% and 2.14%. The pattern for the welding process should stay inside of these limits.

(3) *Statistical limits for the charts on (a) screw machine and (b) electrical characteristic.*

Since these involve a different type of data (individual measurements rather than percentages), it is necessary to use a different equation.

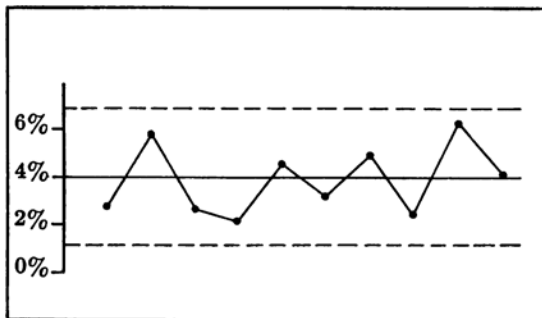


Fig. 5. Control chart for the percentage of loose spoolheads.

Limits of fluctuation around the average of the data = $\pm 2.66 M\bar{R}$,

where the symbol " $M\bar{R}$ " refers to the average difference between successive pairs of measurements. This is explained on page 21.

The limits of fluctuation for the screw machine chart turn out to be .3102 and .3086. The limits of fluctuation for the electrical chart are 1.95 and 0.75. The patterns for these two processes should stay inside of these limits.

In a similar way, limits can be calculated for any other type of data. Detailed instructions are given on pages 12-23.

When we add the statistical limits to a fluctuating pattern, the result is called a "control chart." The control chart is one of the most sensitive devices known for analyzing data and obtaining information.

B-1.5 Meaning of a control chart

The following are control charts for the four operations discussed above.

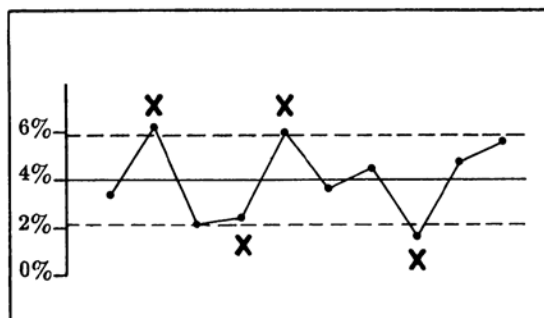


Fig. 6. Control chart for the percentage of offcenter welds.

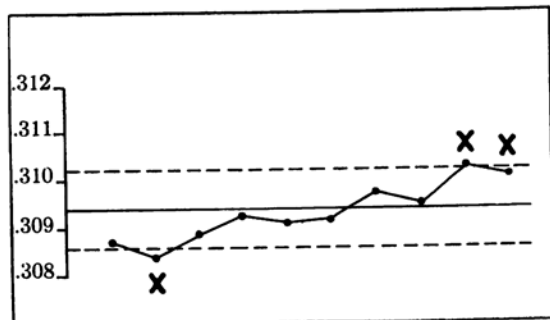


Fig. 7. Control chart for screw machine parts (diameter).

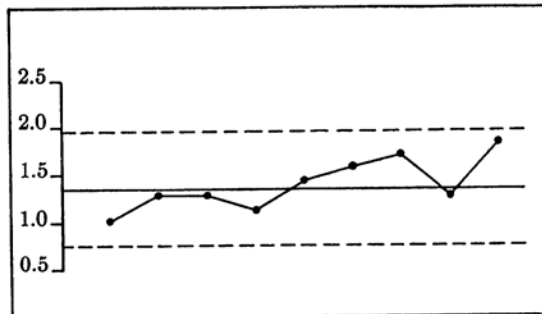


Fig. 8. Control chart for electrical assemblies (noise level).

The statistical limits are drawn in as dotted lines and are called "control limits." The control limits used in this Handbook, unless otherwise stated, are "3 sigma control limits." *

The control limits are used to determine whether the pattern is "natural" or "unnatural." The following procedure is used:

- (1) Check the fluctuating pattern to see whether it is in conflict with the natural statistical limits. The pattern is in conflict if it (a) jumps outside the control limits or (b) forms unnatural clusters of points inside the control limits.

Tests for unnatural points or clusters of points are given on pages 23-28.

- (2) Mark any unnatural points or clusters of points with "x's."
- (3) If the pattern is not in conflict with the limits (that is, there are no x's), consider it a "natural" pattern. In general, the longer the series of points without evidence of unnaturalness, the stronger is the evidence that this is a natural pattern.

An occasional "x" (perhaps once in a hundred points) may be the result of chance alone, and is not considered to make the pattern unnatural.

- (4) If the pattern is in conflict with the limits (that is, there are x's), consider the pattern "unnatural" and the process "out of control." The more numerous the x's, in general, the stronger is the evidence of lack of control.

When a pattern is natural, it means that there are no abnormal extraneous causes working in the process. When the pattern is unnatural, it means that outside disturbances are present and are affecting the process.

When a pattern is unnatural, those familiar with the process should investigate to find what the outside disturbances are.

On most control charts, we prefer to have a longer series of points than those shown in Figures 5-8. In the following discussion these charts should be considered as typical portions of a more complete control chart.

* "Sigma" (usually written σ) is a unit of measure which is used to describe the width or spread of a distribution or pattern. The fluctuations in a "natural" pattern tend to spread about ± 3 sigma.

Interpretation of Figures 5-8

The chart for the spoolhead operation (Figure 5) is interpreted as follows:

- (1) There are no x's.
- (2) There is no evidence that the process is out of control.
- (3) It is not being disturbed by any unusual, outside causes.

The chart for the welding operation (Figure 6) is interpreted as follows:

- (1) Four out of the 10 points are marked with x's.
- (2) There is strong evidence that the process is out of control.
- (3) It is being disturbed by large and unnecessary outside causes.

The chart for the screw machine process (Figure 7) is interpreted as follows:

- (1) Three out of the 10 points are marked with x's.
- (2) There is strong evidence that the process is out of control.
- (3) The pattern shows a continuous movement in one direction which, in the presence of x's, indicates a trend.

The chart for the electrical process (Figure 8) is interpreted as follows:

- (1) There are no x's.
- (2) There is no evidence that the process is out of control.
- (3) The pattern appears to show an upward movement, but since there are no x's there is no reason to believe that this is a trend.

Note how this information depends on having a set of control limits. Without the control limits, Figures 1 and 2 look very much the same. With the control limits added, they look entirely different. The same is true of Figure 3 as compared with Figure 4.

This leads to one of the most fundamental principles in statistical quality control, which may be stated as follows:

To interpret data correctly, we must have (a) a pattern and (b) control limits. This is the same as saying: To interpret data correctly, put it on a control chart.

B-1.6 Experience as the basis for confidence in control charts

No one has to accept the evidence of a control chart on faith. It is always possible to make an investigation to see whether the chart is correct. The following is a record of the investigations actually made in connection with Figures 5-8.

(1) When the people who had the spoolhead operation made a careful check of all the elements in their process, including the materials, piece parts, tools, fixtures, gages and the activities and habits of the operators and machine setters, they were unable to find anything abnormal or out of order. This agreed with the indications of the control chart.

(2) When the people who had the welding operation made a careful check of all the elements in their process, including the materials, piece parts, tools etc., they discovered to their surprise the following two conditions:

a. The method of regulating the voltage supply was not adequate for this job.

b. Two of the machine setters were being careless in dressing the electrodes.

When these two conditions were corrected, the chart came into control. The statistical analysis made by the control chart had revealed the presence of "causes" which the people were not aware of.

(3) The people who had the screw machine process did not believe that the diameter of the parts could change, since the machine was locked at the beginning of the run and the run was too short to be affected by toolwear. However, when they carefully checked the condition of the machine, they discovered worn threads in the locking device. The machine was being "locked" but was not staying locked. When the locking device was repaired, the trend disappeared from the control chart.

(4) The engineers working on the electrical assemblies believed that the noise level of the product was increasing. They said

that this was due to poor housekeeping in the shop. However, when large quantities of product were tested, made at the beginning and end of the period shown, it was found that the noise level was no higher at the end than at the beginning. This agreed with the indications of the control chart.

This has happened in shops and in laboratories many hundreds of times. Time after time, on chart after chart, investigation and experience have proved that the chart is correct. Anyone can verify this for himself by running a few control charts.

The following evidence is also of direct interest to management:

When large numbers of people in a plant begin to act regularly in accordance with control charts, doing what the charts tell them in the details of running their jobs, the plant almost invariably begins to show certain remarkable results:

- Costs are reduced.
- Quality and yields improve rapidly.
- There are major reductions in scrap, rework and the necessary amount of inspection.
- The engineers find their knowledge about the process increasing.
- Experiments are faster and more successful.
- Many design problems vanish.
- Difficult problems having to do with specifications and requirements are solved easily and economically.

All of this is evidence of the reliability of control charts.

B-2 PRINCIPAL KINDS OF CONTROL CHARTS

The control charts shown in Figures 5-8 were based either on percentages or on individual measurements. However, control limits can be applied with equal ease to any other form of data. Some kinds of data are much more sensitive than others in detecting the causes that tend to disturb a process.

The most common kinds of data, listed in order of sensitivity, are as follows:

- (1) Ranges (difference between highest and lowest in a small group of measurements).
- (2) Averages (especially of small groups).
- (3) Percentages (as in the charts for the spoolhead and welding operations)—or counts, which are another form of percentages.
- (4) Individual numbers, such as temperatures, pressures, records of earnings or absences, chemical analyses etc.

- Gradual change in level.
- Instability (abnormally large fluctuations).
- Stratification (abnormally small fluctuations).
- Interactions (two or more variables acting together).
- Systematic variation.
- Tendency of one chart to follow another.

The two most sensitive kinds of data, ranges and averages, are plotted together to form an " \bar{X} and R " chart. This is the most powerful of all control charts for diagnosing production troubles. Next in order come p -charts, which use percentages, and "charts with control limits based on the moving range," which make use of individual numbers.

There are also various substitutes for, or variations of, the p -chart, which use counts instead of percentages. These are known as np -charts, c -charts or u -charts.

\bar{X} and R charts

These are also called "average and range" charts. The \bar{X} is pronounced "X-bar" and means "average of a sample." The R stands for "range," or difference between highest and lowest in a sample. This chart is plotted as a pair of patterns, one for average and one for range. Directions for making an \bar{X} and R chart are given on pages 12–15.

As indicated above, the \bar{X} and R chart is the most sensitive control chart for tracing and identifying causes. The R pattern is read first, and from this it is possible to identify many causes directly. The \bar{X} pattern is read in the light of the R chart, and this makes it possible to identify other causes. Finally, the \bar{X} pattern and R pattern are read jointly, which gives still further information.

Among the patterns which may form on an \bar{X} and R chart are the following:

- Cycles.
- Trends.
- Freaks.
- Mixtures.
- Grouping or "bunching" of measurements.
- Sudden shift in level.

Each type of pattern can be associated with particular causes. The causes which affect the \bar{X} chart are in general different from the causes which affect the R chart. This information, when combined with job knowledge, makes it possible to isolate the factors disturbing the process.

The patterns listed above are fully explained in Engineering Part F.

p -Charts

These are also called "percentage charts" or "attributes charts." The " p " means "proportion." It often represents the proportion of bad compared to total—that is, the "percent defective." Directions for making a p -chart are given on pages 17–20.

The p -chart is the second most sensitive chart for identifying causes. It is less powerful than the \bar{X} and R chart because it has only one pattern, and thus it is not possible to identify causes so accurately. However, the p -chart forms the same kinds of pattern as \bar{X} and R charts (cycles, trends, freaks, gradual change, sudden shift, etc.), and the patterns can be related to one's knowledge of the variables in the process.

The p -chart relies more heavily on job knowledge than the \bar{X} and R chart. It is therefore used mainly in situations where the important causes are known.

Other attributes charts

For information about np -charts, c -charts and u -charts, see pages 20–21.

Charts for individual measurements with control limits based on the "moving range"

These charts are used for individual numbers which cannot easily be formed into samples. The "moving range" refers to the method of

calculating the control limits, which depends on taking successive differences or ranges between the individual observations. Only the individual numbers are plotted, however, not the moving ranges.

This chart is less sensitive than either the \bar{X} and R chart or p -chart, but it forms the same types of pattern and can be interpreted in much the same way. This chart is used wherever it is not practical to use \bar{X} and R charts or p -charts.

Directions are given on pages 21-23.

B-3 \bar{X} AND R CHARTS

B-3.1 Variables measurements

For \bar{X} and R charts it is necessary to have measurements that are capable of showing actual degrees of variation. Measurements of this type are called *variables measurements*. Common examples of variables measurements are length in feet, diameter in inches, resistance in ohms, noise in decibels, weld strength in grams or pounds, etc. Many characteristics which are not ordinarily measured in the shop (cracks, appearance or workmanship, for example) can be put in a form which is roughly equivalent to variables measurements, if necessary. Methods of doing this are explained on pages 49-50.

B-3.2 Definitions and symbols for \bar{X} and R charts

The following symbols are used in connection with \bar{X} and R charts:

X = an individual reading or observation.

n = the number of observations in a group or set, often referred to as the sample size. "n" may be 2, 3, 4, 5 or more, but should not be greater than 10.

\bar{X} (X bar) = the average of a group of X 's. \bar{X} is calculated by the formula

$$\bar{X} = \frac{X_1 + X_2 + X_3 \dots X_n}{n}$$

$\bar{\bar{X}}$ (X double bar) = the average of a series of \bar{X} values.

R (range) = the difference between the larg-

est and smallest reading in a sample of n measurements.

\bar{R} (R bar) = the average of a series of R values.

A_2 = a factor used in calculating the control limits for the \bar{X} chart.

D_4 = a factor used in calculating the upper control limit for the R chart.

D_3 = a factor used in calculating the lower control limit for the R chart.

The factors A_2 , D_4 and D_3 vary with the size of the sample. Select the appropriate factor from the table shown below.

Number of Observations in Sample (n)	\bar{X} Chart. Factors for Control Limits	R Chart. Factors for Control Limits	
	(A_2)	(D_3)	(D_4)
2	1.88	0	3.27
3	1.02	0	2.57
4	.73	0	2.28
5	.58	0	2.11
6	.48	0	2.00
7	.42	.08	1.92
8	.37	.14	1.86
9	.34	.18	1.82
10	.31	.22	1.78

Fig. 9. Table of factors for \bar{X} and R charts.

B-3.3 Directions for making an \bar{X} and R chart

Always make the R chart before the \bar{X} chart. Proceed as follows.

R chart

- (1) Decide on the sample size (n) to be used.
- (2) Obtain a series of groups of measurements, each group containing "n" measurements. Have 20 or more groups, if possible, but not less than 10 groups.
- (3) Compute R for each sample and take the average of the R 's (\bar{R}). This is the centerline for the R chart. It is drawn as a solid horizontal line. See Figure 10.
- (4) Multiply \bar{R} by D_4 and D_3 to obtain upper and lower control limits for the R chart.

The control limits are drawn as dotted horizontal lines. See Figure 10.

- (5) Obtain a piece of graph paper (or a standard control chart form) and set up an appropriate scale. Be careful not to make the R chart too wide. Set up the R chart at the bottom of the sheet as shown in Figure 10.

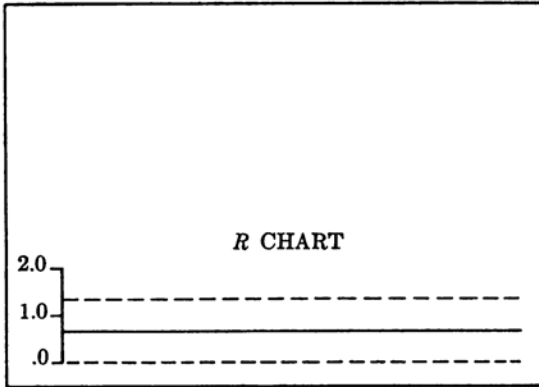


Fig. 10. Construction of the R chart.

- (6) Plot on this chart the successive values of R and connect the points with straight lines.
- (7) Mark x's on the chart, if necessary, in accordance with the rules given on pages 25-28.
- (8) Interpret the chart as explained on page 30. For a fuller interpretation of the R chart see pages 154-156.

\bar{X} chart

- (1) Use exactly the same groups of measurements that were used for the R chart.
- (2) Calculate \bar{X} for each sample and take the average of the \bar{X} 's ($\bar{\bar{X}}$). This is the centerline for the \bar{X} chart. It is drawn as a solid horizontal line. See Figure 11.
- (3) Multiply \bar{R} by A_2 to get the width of the control limits for the \bar{X} chart. Add the $A_2\bar{R}$ value to (and subtract it from) $\bar{\bar{X}}$ to get the location of the control limits.

Upper control limit for \bar{X} chart = $\bar{\bar{X}} + A_2\bar{R}$.

Lower control limit for \bar{X} chart = $\bar{\bar{X}} - A_2\bar{R}$.

The control limits are shown as dotted horizontal lines. See Figure 11.

- (4) Choose a scale for the \bar{X} chart that is properly related to the scale already chosen for the R chart. The scale should be such that the distance between the control limits on the \bar{X} chart is roughly similar to the distance between the control limits on the R chart. This relationship can be obtained as follows:

- a. For samples of 2, use the same spacing on the \bar{X} chart as on the R chart.
- b. For samples of 5, let each division on the graph paper represent an increment of measurement half as large as on the R chart.
- c. For samples of 10, let each division on the graph paper represent an increment of measurement one-third as large as on the R chart.

This relationship between the scales corresponds roughly to $1/\sqrt{n}$. For other sample sizes, use the scale nearest to those given above.

- (5) Set up an \bar{X} chart in the upper portion of the sheet as shown in Figure 11.

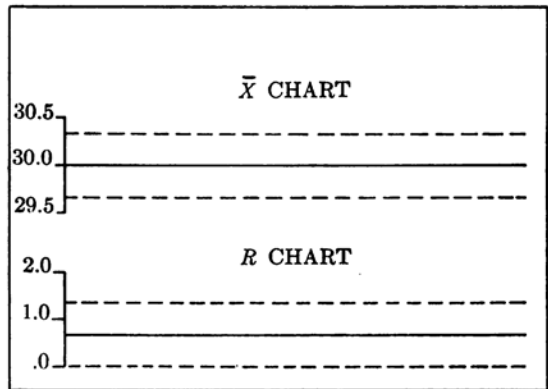


Fig. 11. Construction of the \bar{X} chart.

- (6) Plot on this chart the successive values of \bar{X} and connect the points with straight lines.
- (7) Mark x's on the chart, if necessary, in accordance with the rules given on pages 25-28.
- (8) If drawing limits are specified for the characteristic being plotted, draw arrows in the left-hand margin along the \bar{X} scale to represent the drawing limits. See

page 30. Notice how the arrows are drawn in Figure 45 on page 31.

- (9) Interpret the chart as explained on pages 30-31. For a fuller interpretation of the \bar{X} chart, see pages 152-154 and 156.

Directions for samples of 5

Since samples of 5 are used more commonly than any others, the instructions for this sample size are summarized for easy reference in Figure 12.

\bar{X} and R chart for n = 5

Divide the data into groups of 5.

R Chart

For each group obtain R .
Centerline on R chart = \bar{R} .
Lower control limit = 0.
Upper control limit = $2.11 \bar{R}$.

\bar{X} Chart

For each group obtain \bar{X} .
Centerline on \bar{X} chart = $\bar{\bar{X}}$.
Control limits = $\bar{\bar{X}} \pm .58 \bar{R}$

Fig. 12. Instructions for \bar{X} and R chart—samples of 5.

B-3.4 Example of calculations for an \bar{X} and R chart

Obtain a set of data as shown in Figure 13.

- (1) Centerline for R chart:

$$\bar{R} = \frac{\text{Total of } R\text{'s}}{\text{No. of Samples}} = \frac{31.8}{20} = 1.59$$

- (2) Control limits for R chart:

$$D_4\bar{R} = 2.11 \times 1.59 = 3.35$$

$$D_3\bar{R} = 0 \times 1.59 = 0$$

- (3) Centerline for \bar{X} chart:

$$\bar{\bar{X}} = \frac{\text{Total of } \bar{X}\text{'s}}{\text{No. of Samples}} = \frac{213.20}{20} = 10.66$$

- (4) Control limits for \bar{X} chart:

$$\bar{\bar{X}} + A_2\bar{R} = 10.66 + (.58 \times 1.59)$$

$$= 10.66 + .92 = 11.58$$

$$\bar{\bar{X}} - A_2\bar{R} = 10.66 - .92 = 9.74$$

The completed chart is shown in Figure 14.

GAIN IN DB.										
	3/31			4/1			4/4		4/5	
	1	2	3	4	5	6	7	8	10	
	11.1	9.6	9.7	10.1	12.4	10.1	11.0	11.2	10.6	8.3
	9.4	10.8	10.0	8.4	10.0	10.2	11.5	10.0	10.4	10.2
	11.2	10.1	10.0	10.2	10.7	10.2	11.8	10.9	10.5	9.8
	10.4	10.8	9.8	9.4	10.1	11.2	11.0	11.2	10.5	9.5
	10.1	11.0	10.4	11.0	11.3	10.1	11.3	11.0	10.9	9.8
	52.2	52.3	49.9	49.1	54.5	51.8	56.6	54.3	52.9	47.6
\bar{X}	10.44	10.46	9.98	9.82	10.90	10.36	11.32	10.86	10.58	9.52
R	1.8	1.4	.7	2.6	2.4	1.1	.8	1.2	.5	1.9
			4/6			4/7		4/8		
	11	12	13	14	15	16	17	18	19	20
	10.6	10.8	10.7	11.3	11.4	10.1	10.7	11.9	10.8	12.4
	9.9	10.2	10.7	11.4	11.2	10.1	12.8	11.9	12.1	11.1
	10.7	10.5	10.8	10.4	11.4	9.7	11.2	11.6	11.8	10.8
	10.2	8.4	8.6	10.6	10.1	9.8	11.2	12.4	9.4	11.0
	11.4	9.9	11.4	11.1	11.6	10.5	11.3	11.4	11.6	11.9
	52.8	49.8	52.2	54.8	55.7	50.2	57.2	59.2	55.7	57.2
\bar{X}	10.56	9.96	10.44	10.96	11.14	10.04	11.44	11.84	11.14	11.44
R	1.5	2.4	2.8	1.0	1.5	.8	2.1	1.0	2.7	1.6

Fig. 13. Typical data for an \bar{X} and R chart.

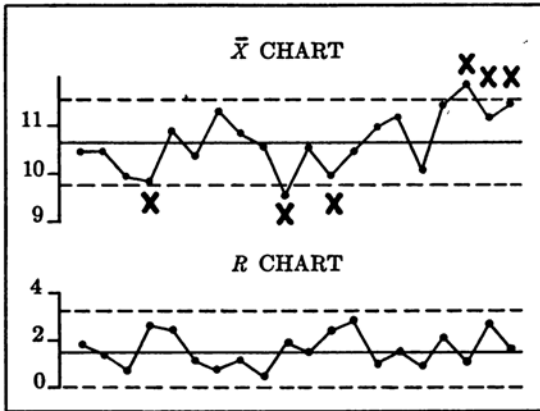


Fig. 14. Example of a finished \bar{X} and R chart.

B-3.5 Example of the use of an \bar{X} and R chart

A certain shop was having trouble in meeting a dimensional specification. The engineer decided to take a sample of the parts coming from the process and group them together in the form of a frequency distribution. The result of taking parts from a tote pan, measuring them and plotting the distribution is shown in Figure 15.

This distribution showed that parts were failing to meet the specification on both the high and low sides. No apparent solution was found, however, by treating the data in this manner. The engineer then made a second attempt to analyze the trouble as follows:

These parts were being produced by two machines, each of which was supposed to perform the same function. The engineer decided to measure parts from the individual machines.

The separate distributions are shown in Figure 16.

This also failed to give a solution to the problem. Both machines were in trouble and the distributions were very similar.

At this point the engineer decided to plot an \bar{X} and R chart. This required grouping the data into samples of 5 for each machine and plotting the average and range of each group of measurements in the order in which they were produced. The result of this is shown in Figures 17 and 18 on page 16.

These charts showed immediately that there was a significant difference between the two machines. Machine No. 1 had a wide pattern on its R chart (Figure 17) and the pattern was not in control. This meant that the machine was not behaving consistently. There was something about it that needed maintenance or repair.

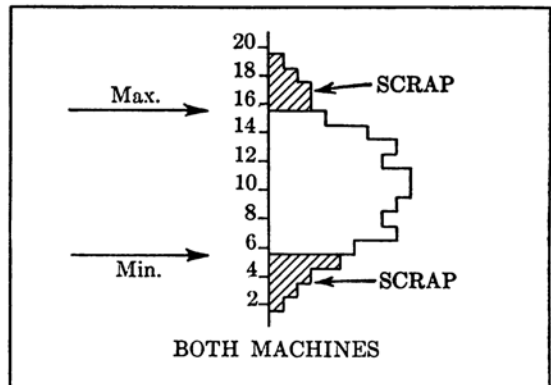


Fig. 15. Typical distribution from a process which is in trouble.

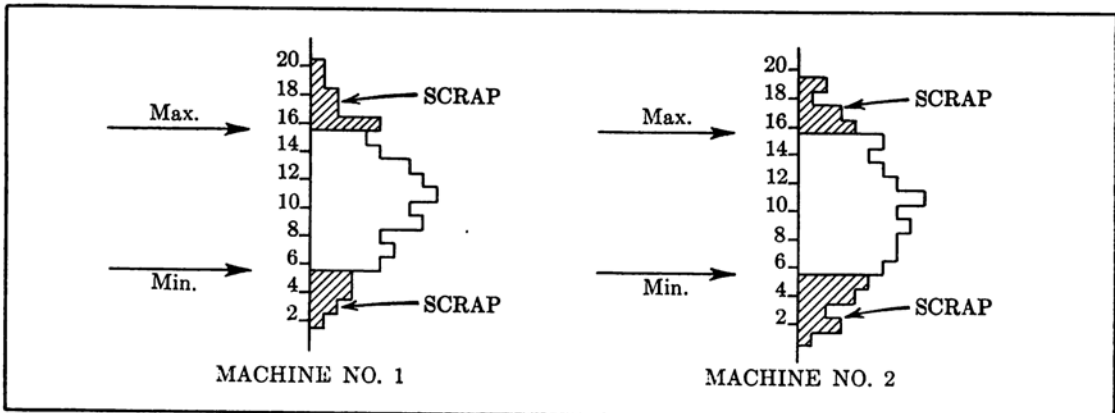


Fig. 16. Separate distributions from individual machines.

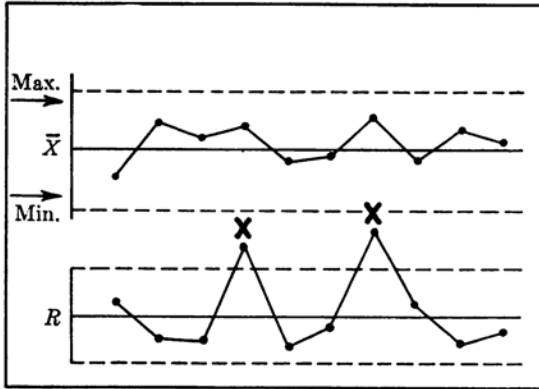


Fig. 17. \bar{X} and R chart for Machine No. 1.

When the Maintenance Department checked Machine No. 1 they found a worn bearing. When this was replaced, the R chart became much narrower and the product was no longer in trouble.

The cause of the trouble on Machine No. 2 was entirely different. This machine was already in good repair, as shown by the controlled pattern on its R chart.

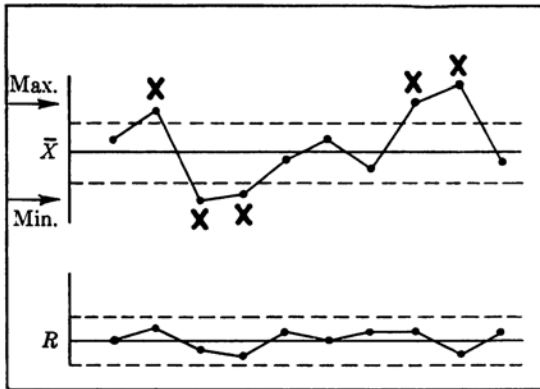


Fig. 18. \bar{X} and R chart for Machine No. 2.

However, the center of the distribution on this machine was shifting back and forth, from the high side to the low side. This is shown by the out-of-control pattern on its \bar{X} chart. This meant that the machine setter was either making poor settings or changing the setting too often.

When the machine setter was instructed to stop re-setting the machine on the basis of one or two measurements, and to use a control chart instead, the trouble disappeared also from Machine No. 2.

The \bar{X} and R chart is able to separate causes of trouble on processes of all types, operator-controlled as well as machine-controlled.

B-3.6 Additional information about \bar{X} and R charts

- (1) By studying the \bar{X} chart and the R chart together, it is possible to get indications of whether the distribution of product is symmetrical or skewed. See page 156.
- (2) By studying the \bar{X} chart and R chart together, it is possible to tell whether the process is capable of meeting a specification. This is explained briefly on pages 30-31. More detailed information is given on pages 56-61.
- (3) Do not confuse the control limits on the \bar{X} chart with the specified "blueprint" limits which are commonly found on a drawing. The control limits are natural process limits, determined by calculation from the nature of the process itself. The limits on a drawing are artificial, selected limits. They may or may not have any reasonable connection with the natural limits of the process. One of the objects of the control chart is to see how the natural limits compare with the limits shown on the drawing.

Also, the control limits on an \bar{X} chart are limits for *averages*, while the limits on a drawing are usually limits for *individuals*. Limits for averages are always narrower than limits for individuals.

B-3.7 Useful adjuncts to the \bar{X} and R chart

The data which have been used to plot an \bar{X} and R chart may also be plotted in two other ways:

- a. As a running record of individual measurements.
- b. As a frequency distribution.

While neither of these should be used in place of an \bar{X} and R chart, they are both very useful when plotted in *addition*. They give information which reinforces the \bar{X} and R chart.

B-4 p-CHARTS AND OTHER ATTRIBUTES CHARTS

B-4.1 Attributes measurements

The p -chart does not require any actual measurements such as a measurement of length, width or resistance. It is only necessary to count the number of pieces that are oversize or warped, or have any other characteristic in which we are interested, and translate this count into a percentage. Measurements of this type which depend on counting are called *attributes measurements*. Common examples of attributes measurements are the number of cracked insulators in a lot, the number of poor soldered connections in a piece of wired equipment, the percent defective in the work of a certain operator, etc.

p -Charts are used in places where it is difficult or uneconomical to make a numerical measurement, or where it is desired to combine many types of defect into an overall percentage. However, if many characteristics are combined on a single chart, the p -chart becomes progressively more difficult to interpret.

B-4.2 Definitions and symbols for a p-chart

The following symbols are used in connection with p -charts:

n = number of units in a sample.

p = fraction defective in a sample.

$$= \frac{\text{number of defective units in a sample}}{\text{sample size } (n)}$$

\bar{p} = average fraction defective in a series of samples

$$= \frac{\text{total number of defective units in all samples in the series}}{\text{total number checked in all samples in the series}}$$

B-4.3 Directions for making a p-chart

Proceed as follows:

- (1) Obtain a series of samples of some appropriate size. Convenient sample sizes are 50 and 100. The "sample" may actually

be the complete lot if the entire lot has been checked. Have 20 or more groups if possible, but not less than 10 groups.

- (2) Count the number of defective units (warped, undersize, oversize, or whatever the characteristic may be in which you are interested). Calculate the value of p for each sample.
- (3) Calculate \bar{p} (the average fraction defective) as shown in B-4.2. This is the centerline for the p -chart. It is drawn as a solid horizontal line. See Figure 19.
- (4) Calculate upper and lower control limits for the p -chart by using the following formulas.

Upper control limit for p -chart =

$$\bar{p} + 3 \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

Lower control limit for p -chart =

$$\bar{p} - 3 \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

The control limits are drawn as dotted horizontal lines. See Figure 19.

- (5) Decide on an appropriate scale and set up a p -chart on graph paper or a standard control chart form. Be careful not to make the p -chart too wide. See Figures 19 and 21.

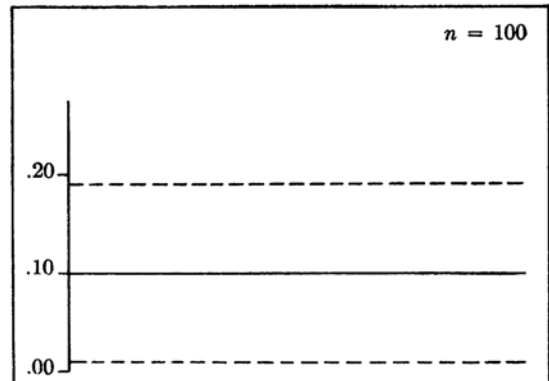


Fig. 19. Construction of a p -chart.

- (6) Plot on this chart the successive values of p and connect the points with straight lines.
- (7) Mark x's on the chart, if necessary, in accordance with the rules given on pages 25-28.
- (8) Interpret the chart as explained on page 31. For a fuller interpretation of the p -chart, see pages 157-159.

Average value of n

If the samples to be used for a p -chart are not of the same size, it is sometimes permissible to use the *average sample size* for the series in calculating the control limits. The largest sample in the series should not be more than twice the average sample size, and the smallest sample in the series should not be less than half the average sample size.

If the individual samples vary more than this, either separate or combine samples to make them of suitable size, or calculate control limits separately for the samples which are too large or too small. See paragraph B-4.5.

p -Charts are most useful when this problem is avoided by keeping the sample size constant.

B-4.4 Example of calculations for a p -chart

Obtain a set of data as shown in Figure 20.

- (1) Centerline for p -chart:

$$\bar{p} = \frac{\text{total number of defective units in all samples}}{\text{total number checked in all samples}}$$

$$= \frac{2103}{36060} = .0583, \text{ or } 5.83\%$$

- (2) Average n for p -chart:

$$\text{average } n = \frac{\text{total number checked in all samples}}{\text{number of samples in the series}}$$

$$= \frac{36060}{25}$$

$$= 1442$$

- (3) Upper control limit for p -chart:

$$= \bar{p} + 3 \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

$$= .0583 + 3 \sqrt{\frac{.06 (.94)}{1442}}$$

$$= .0583 + .0186 = .0769$$

Lower control limit for p -chart

$$= .0583 - .0186 = .0397.$$

These may be converted to percentage by multiplying by 100. The completed chart is shown in Figure 21.

Separate control limits should properly be calculated for the samples dated 9/12 and 9/15.

B-4.5 Stairstep limits on a p -chart

The control limits in Figure 21 were calculated by using an average value of n . It is also possible to calculate control limits for each point on a p -chart individually, using the in-

Date	Number in Sample	Number Defective in Sample	% Defective in Sample
8/11	1524	70	4.59
8/12	1275	53	4.16
8/15	1821	132	7.25
8/16	1496	91	6.08
8/17	1213	32	2.64
8/18	1371	55	4.01
8/19	1248	69	5.53
8/20	1123	67	5.97
8/22	1517	159	10.48
8/23	1488	94	6.32
8/24	2052	105	5.12
8/25	1696	37	2.18
8/26	1427	58	4.06
8/29	1277	75	5.87
8/30	1613	73	4.53
9/2	1987	145	7.30
9/6	1360	41	3.01
9/7	1439	50	3.47
9/8	1723	118	6.85
9/9	2035	169	8.30
9/10	1314	88	6.70
9/12	215	24	11.16
9/13	1384	77	5.56
9/14	1995	185	9.27
9/15	467	36	7.71
Total	36060	2103	

Fig. 20. Typical data for a p -chart.

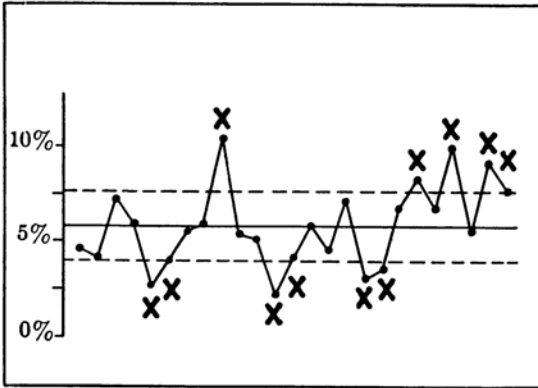


Fig. 21. Example of a finished *p*-chart.

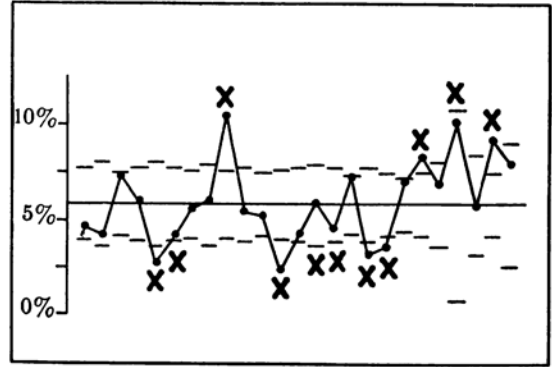


Fig. 22. Stairstep limits for the *p*-chart shown in Fig. 21.

dividual values of n . When this is done, the control limits may have an irregular "stair-step" effect as shown in Figure 22. Where the sample size is small, the control limits are wider. Where the sample size is large, the control limits are tighter. The pattern is marked with x 's in the same way as if these were ordinary control limits.

"Stairstep" limits make a *p*-chart more difficult to interpret, and their use should be avoided. As mentioned above, limits of this kind are occasionally used for individual samples, where the sample is too small or too large to be covered by the "average value of n ."

B-4.6 Example of the use of *p*-charts

In the manufacture of a relay, 24 palladium contacts are welded to the tips of 24 wire springs. One of the desired quality characteristics of the relay is the proper location of the contacts on the springs. The print tolerance allows for a maximum of .010" offcenter. That is, the centerline of the contact must not be more than .010" to the left or right of the centerline of the wire. The automatic welder designed for this operation has two welding heads. The first head welds the odd-numbered contacts and the second head welds the even-numbered contacts.

In the process of proving-in the welder, a *p*-chart was made for each head. After four days of operation the *p*-charts indicated two different levels, 10% defective on the first head and 22% defective on the second head. The second head also indicated an out-of-control condition.

An investigation was made on the feeding and guiding mechanism for each head. Replacement of the contact wire guides on both heads, and a tightened wire comb and locator on the second head, brought the percent defective down to a controlled average of 3% for both heads.

The process was allowed to continue at this level and the *p*-charts were continued on a 100% inspection basis. Occasional out-of-control points were noticed on each head due to loosening of the wire comb and locator. A redesign of the wire comb and locator to prevent its working loose and shifting led to a stabilized process at an average level of 0.5% defective. This level was acceptable to the assembly department.

The *p*-charts were continued, but put on a sampling basis. The charts acted as warning devices on the wearing or loosening of the guides and locators. Figure 23 shows the

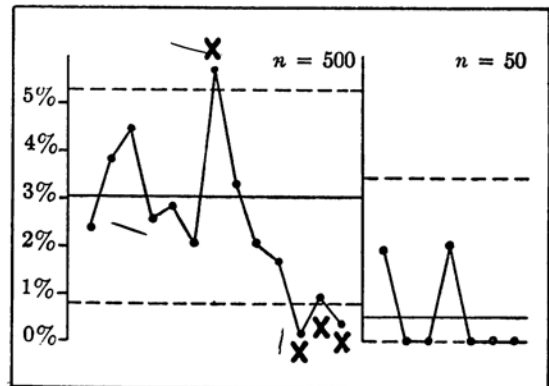


Fig. 23. Improvement resulting from a *p*-chart.

reduction in percent defective which was brought about by the effective use of p -charts.

B-4.7 Additional information about p -charts

- (1) When the lower control limit for a p -chart is found to be less than zero, it should be shown as zero on the chart.
- (2) In setting up the chart and plotting the samples, it is permissible to work with either decimals or percentages. On shop charts it is generally preferable to use percentages.

In calculating the control limits for a p -chart, however, always use the decimal value instead of the percentage. The limits may then be converted to percentages after the calculations are completed.

B-4.8 np -Charts

If the samples to be plotted on a p -chart are all of the same size, it is simpler to plot the *number of defectives* found in each sample instead of calculating the percentage. When this is done, the chart is called an np -chart instead of a p -chart.

Control limits for the np -chart are calculated as follows:

For each of a series of equal samples, count the number of defectives, np .

Centerline on chart = $n\bar{p}$

Control limits = $n\bar{p} \pm 3\sqrt{n\bar{p}(1-\bar{p})}$

Except for plotting the number defective instead of the percentage or fraction, this chart is in every way the equivalent of a p -chart. Plot the points, connect them, mark the x 's and interpret the pattern in the same way as on a p -chart.

B-4.9 c -Charts

The c -chart is a special type of attributes control chart which uses the number of *defects* instead of the number of *defectives*. The distinction between "defect" and "defective" is as follows:

A defect is an individual failure to meet a single requirement.

A defective is a unit of product which contains one or more defects.

It is possible for a unit of product to contain many "defects" and still be counted as only one "defective."

In cases where a single unit of product is likely to contain many defects, a c -chart is generally more appropriate than a p -chart.

Nature of the c -chart : theoretical restrictions

The c -chart may be considered a special form of p -chart, in which

- (a) the possibilities for defects are theoretically infinite, and
- (b) the probability of getting a defect at any specific point is very small.

With a complicated product, the number of possibilities for defects increases very rapidly, until it approaches (for practical purposes) an "infinite" number of possibilities. Under these circumstances it is generally possible to use a c -chart.

c -Charts can be used only where the "area of opportunity for finding defects" is kept constant. That is, the samples must all consist of a certain fixed length, area, quantity, etc. If the first sample consists of 10,000 conductor-feet of wire, do not let the second sample consist of 20,000 conductor-feet, or the third sample of 8000. If the sample is to be a square yard of paper or other material, each successive sample must be of the same size.

The sample may, however, consist of a constant number of units considered as if they were one.

Typical places for using c -charts are the following:

- Number of loose soldered connections on a piece of wired equipment.
- Number of flaws in a square foot of rubber or cloth.
- Number of foreign particles inside of a sealed switch.
- Number of defects of all kinds on a crossbar frame.
- Number of points at which breakdown occurs per thousand feet of insulated wire.
- Number of reported failures in a radar system per month.

Directions for plotting

Control limits for the c -chart are calculated as follows:

For each of a series of equal samples, count the number of defects, c .

$$\text{Centerline on chart} = \bar{c}$$

$$\text{Control limits} = \bar{c} \pm 3\sqrt{\bar{c}}$$

c -Charts are plotted, marked and interpreted in the same way as p -charts. An example of a c -chart is given on pages 72-73.

B-4.10 u -Charts

The u -chart is a variation of the c -chart. The point plotted is the average number of defects per unit in a sample of n units. Each of the units in the sample must satisfy the requirements for a c -chart, but the number of units which are averaged need not be the same for all samples. This type of chart is sometimes used for aircraft, radar systems and other complicated assemblies.

Control limits for the u -chart are calculated as follows:

$$u = \frac{\text{number of defects}}{n \text{ (number of equal units)}}$$

$$\text{Centerline on chart} = \bar{u}$$

$$\text{Control limits} = \bar{u} \pm 3\sqrt{\frac{\bar{u}}{n}}$$

If the number of units is not the same from sample to sample, handle the varying sample sizes in the same manner as for a p -chart. u -Charts are plotted, marked and interpreted in the same way as p -charts or c -charts.

B-5 CHARTS FOR INDIVIDUAL MEASUREMENTS WITH CONTROL LIMITS BASED ON THE MOVING RANGE

B-5.1 Individual measurements

The principal kinds of data for which this chart should be used are the following:

- a. Accounting figures of all kinds, including shipments, efficiencies, absences, losses,

inspection ratios, maintenance costs, accident reports, records of medical tests, etc.

- b. Production data such as temperatures, pressures, voltages, humidity, conductivity, furnace heat, gas composition, the results of chemical analysis, etc.

In all of these cases only one number is available to represent a given condition.

B-5.2 Definitions and symbols for charts with "moving range" limits

The only new symbol is MR which stands for the "moving range." This is the difference between successive pairs of numbers in a series of numbers: that is, the difference between the first and second numbers, then between the second and third, then between the third and fourth, etc.

Each of the individual numbers is used in calculating two of the moving ranges.

$M\bar{R}$ is the average of a series of moving ranges.

B-5.3 Directions for making a chart with "moving range" limits

To plot this type of chart, proceed as follows:

- (1) Start with a series of individual numbers. Have 20 or more numbers if possible, but not less than 10 numbers.
- (2) Take the difference between the first and second numbers, and record it; then the difference between the second and third numbers, etc. Continue in this way until you have taken the difference between the next-to-the-last and the last numbers. The number of differences or "ranges" should be one less than the number of individuals in the series.

The differences are calculated without regard to sign. That is, it does not matter whether they are plus or minus (gain or loss).

- (3) Take the average of the original numbers in the series (\bar{X}). This is the centerline for the chart. It is drawn as a solid horizontal line. See Figure 25 (page 22).
- (4) Take the average of the "ranges" obtained in step (2). Be sure to divide by